The Enhanced Indispensability Argument: Representational vs. Explanatory Role of Mathematics in Science

Juha Saatsi

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Abstract

The Enhanced Indispensability Argument (Baker [2009]) exemplifies the new wave of the indispensability argument for mathematical Platonism. The new wave capitalises on mathematics' role in scientific explanations. I will criticise some analyses of mathematics' explanatory function. In turn I will emphasise the *representational* role of mathematics, and argue that the debate would significantly benefit from acknowledging this alternative viewpoint to mathematics' contribution to scientific explanations and knowledge.

1 Introduction

The Enhanced Indispensability Argument (Baker [2009]) exemplifies the new wave of the indispensability argument for mathematical Platonism. The new wave capitalises on mathematics' role in scientific explanations. I will criticise some analyses of mathematics' explanatory function. In turn I will emphasise the *repres-*

entational role of mathematics, and argue that the debate would significantly benefit from acknowledging this alternative viewpoint to mathematics' contribution to scientific explanations and knowledge.

The Enhanced Indispensability Argument runs as follows (Baker [2009]):

(1) We ought rationally to believe in the existence of any entity which plays an indispensable explanatory role in our best scientific theories.

(2) Mathematical objects play an indispensable explanatory role in science.

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(3) Hence, we ought rationally to believe in the existence of mathematical objects.

This argument allegedly enhances the original one by virtue of (i) focusing on the indispensable *explanatory* role of mathematics—not just on indispensability *simpliciter*—and (ii) appealing to the role of inference to the best explanation in the defense of scientific realism. (Although (ii) is not explicit in the argument as given above, its role in motivating (1) should be transparent.) The basic idea simply is that since mathematical entities arguably feature in our best scientific explanations on a par with unobservable entities, and since rational belief in the unobservable entities is defended via inference to the best explanation vis-à-vis scientific explanations, the realist should also be a Platonist regarding the explanatory bits of mathematics.

EIA provides an interesting and novel spin on the sort indispensability of mathematics that can be used to argue for Platonism on the basis of the undeniable inseparability of applied mathematics and science. But it faces severe challenges. Most of the challenges in the literature have focused on the premise (2) above, questioning the claim that mathematics plays *right kind of role*. Baker bravely defends this premise against these challenges, but some serious worries that remain. I will explore these below.

2 Representation vs. explanation

In order to brace the second premise of EIA the proponents of the argument have put forward several examples of mathematics playing an indispensable explanatory role. The examples mentioned in Baker ([2009]) vary from geometrical explanations in physics to number-theoretic explanations of the evolution of the life-cycles of the North-American cicada. My general worry about all of these examples is that they do not separate with sufficient clarity the *explanatory* from the *representational* role of mathematics.[1] I admit that the cases discussed in the literature demonstrate that mathematics can play a knowledge-conferring role in science: it can help us learn about the world. But the fact that mathematics can give us knowledge (or better justified beliefs) of certain physical facts does not automatically entail that it thereby plays an explanatory role.

Mathematics can help us learn about the world by virtue of playing a representational (and, derivatively, inferential and justificatory) role in science. The fact that mathematics can play such a role is something that calls for an explanation. But *this* issue, and any argument for Platonism that turns on this issue,

[1] Melia ([2000]) voices a related worry by contrasting the *expressive* capacity of mathematics with its capacity to *simplify* our world view. Melia ([2002]) separates representational and explanatory roles of mathematics with respect to geometrical explanations. Here I aim to throw further light on the representational virtues of mathematics.

is independent from EIA. The latter concerns not the representational capacity of mathematics in general, but more specifically the capacity and role that mathematics *per se* has in explaining and yielding understanding about the concrete world. I now claim that by carefully attending to the kind of representational role that mathematics can play, we find that in the examples mentioned in Baker ([2009]) mathematics *per se* only plays a representational role, and mathematics need not be viewed as being explanatory of the physical. I'll next revisit some of the key examples to show why this is the case.

2.1 Honeycomb.

Why is hive-bee honeycomb always divided up into hexagons instead of some other polygons (or some combination thereof)? According to Lyon & Colyvan ([2008]) mathematics plays an explanatory role here:

> [T]he biological part of the explanation is that those bees which minimise the amount of wax they use to build their combs tend to be selected over bees that waste energy by building combs with excessive amounts of wax. The mathematical part of the explanation then comes from what is known as the honeycomb conjecture: a hexagonal grid represents the best way to divide a surface into regions of equal area with the least total perimeter. (Lyon & Colyvan [2008], p. 230)

It is not so clear that the explanation for the hexagonal structure of honeycomb is something that can be thus divided into biological and mathematical parts which are both bona fide explanatory. We haven't been given any philosophical account of explanation to support the argument, so it is difficult to say. All we have is an intuition that mathematics is *somehow* relevant for knowing why the bees build their honeycomb the way they do. I will now argue that mathematics' relevance has to do with the way it helps us to know physical explanatory facts, and that playing *this* role doesn't render mathematics explanatory in itself.

My preferred analysis of mathematics' role here goes as follows. Instead of speaking of the 'mathematical part' of the explanation we can say that (a) mathematics can be used to *represent* all physically possible spatial configurations of honeycomb walls, and (b) by virtue of this representational capacity we can *infer* that the area of honeycomb walls is minimised with hexagonal cells. What we infer is a *physical fact*: in Euclidean physical space the hexagonal construction yields the minimum wall area. Mathematics may be indispensable for getting to know this crucial physical fact to which the full evolutionary explanation of the phenomenon then appeals. There is no "mathematical part" to this evolutionary explanation. Rather, mathematics only plays a role in representing physical facts concerning areas and volumes in Euclidean space, allowing us to infer certain physical facts from other physical facts, and hence providing us knowledge of the crucial explanatory physical fact.

I can envisage two responses to this attempt to deny the explanatory value of Hales's ([2001]) proof of the honeycomb conjecture. One might suggest, first of all, that before the famous proof we simply couldn't know the relevant explanatory physical fact, and hence we couldn't have an explanation. That is, mathematics is explanatory *by virtue of* providing us knowledge of the crucial physical fact.

This suggestions doesn't seem right. Mathematics can be knowledge-conferring by virtue of providing *justification* for a hypothesis regarding a physical fact without explaining that fact. It can do this by demonstrating that the fact in question "follows" from other physical facts regarding which we have better justified beliefs.[2] This demonstrative-cum-justificatory function of mathematics relies on its representational role. In as far as Euclidean geometry in mathematics correctly represents the structure of physical space, we can use the former to infer facts about the latter, just as we can use an accurate map to infer facts about surrounding topography, say. But facts about physical space are not thereby explained by mathematics, any more than facts about surrounding topography are explained by the map.

Due to explanatory regress we can, of course, ask what explains the explanatory physical fact itself. This takes us to the second possible response to my denial of the explanatory worth of Hales's proof. Now the question is: what explains the fact that hexagonal cells minimise the cell wall area? And allegedly the answer is: Hales's proof.

I think we should not capitulate to explanatory regress here and try thus extending the evolutionary explanation. It may be the case that mathematicians regard Hales's proof as explanatory of the relevant mathematical fact that correctly represents the physical explanans. That is, Hales's proof may well constitute an explanation internal to mathematics.[3] Be that as it may, it doesn't follow from this that the corresponding fact about physical geometry (represented by the mathematics) is explained by the proof. A mathematical proof (explanatory or otherwise) proceeds from a set of axioms that state basic assumptions about the mathematical domain in question. It may be that every axiom required by Hales's proof represents some physical fact. So does the theorem the proof ultimately arrives

[2] I put 'follows' in scare quotes to indicate that I don't wish to imply that some of the physical facts are metaphysically primary in some sense.

[3] Cf. Mancosu ([2008]) for discussion on mathematical explanation in general.

at. But the logical deduction of the theorem from the axioms need not capture any explanatory relation between the facts represented by the theorem and the axioms, respectively. On a casual inspection of the proof it transpires that the facts represented by the axioms are not *causally responsible* for the explanandum; or somehow *constitutive* of the explanandum; or in any way *unifying* the explanandum. Furthermore, given that logical deduction can arguably get the order of explanation simply wrong (as the famous flag pole shows), surely the onus is on the advocate of EIA to provide a theory of explanation that supports the claim that Hales's proof really is bona fide explanatory of a physical phenomenon.

If the mathematical proof isn't explanatory of the physical fact that underwrites the evolutionary explanation, then what is? I don't know. But it seems to me that whatever it is—if there indeed is an explanation for it at all—it is likely to be some metaphysical fact concerning the structure of space. To my mind such metaphysics goes beyond what is required by a successful scientific explanation of the honeycomb phenomenon. Biologists and scientific realists can take as their starting point the assumption that the structure of physical space is accurately (enough) represented by Euclidean geometry, and subject to that assumption mathematical demonstrations such as Hales's are simply a way of getting to know explanatory truths about the structure of physical space either by discovering unexpected facts or by confirming hypotheses that are already expected on purely inductive grounds to be nothing but true.

Consider the hypothesis (going back over two millenia) that the hexagonal configuration minimises the surface area in a honeycomb-like physical structure. This hypothesis, together with the Darwinian evolutionary story, provides more or less complete explanation of this feature of honeycombs. What exactly does

Hales's proof add to this? I would argue that it alters our *justification* for the explanatory hypothesis. This hypothesis concerns an explanatory physical fact for which we had only inductive justification before Hales. The proof of the honeycomb theorem then demonstrated the explanatory physical fact beyond doubt (subject to the assumption that physical space is accurately represented by Euclidean geometry) by using the representational capacity of mathematics. But by virtue doing this the proof does not supplement or change the explanation itself. Nevertheless, the proof is epistemically valuable: we can now more firmly believe in the evolutionary explanation, in as far as we have evidence for Euclidean geometry being suitably accurate representation of the structure of physical space.[4]

The following analogy helps to strengthen the key point. The alleged role of the honeycomb theorem in Lyon and Colyvan's account is essentially no different from using Pythagoras' theorem to "explain" the fact that the hypotenuse length squared is always the sum of the legs' lengths squared for any right triangle in Euclidean physical space (call this the 'Pythagorean character' of physical space). This fact about physical space can play a role in scientific explanations, and we can be well justified in believing this fact on purely inductive grounds, based on a sufficient number of measurements and "geometric experiments", say.[5] Although our knowledge regarding this fact about the physical geometry can have extremely good inductive warrant, it of course cannot match a deductive demonstration from premises which are even better justified. If our beliefs in those properties of phys-

[4]How much justification ensues from the mathematical proof? It is difficult to gauge this. It could be argued that there was a good deal of inductive support for the explanation prior to the proof, by using inference to the best explanation regarding the honeybee's behaviour, for example. Furthermore, since the physical space isn't *exactly* Euclidean, Hales proof doesn't *exactly* apply to real honeycombs.

[5]I am thinking of the various pictorial "proofs" of Pythagoras' theorem. Cf. Nelsen ([1997])

ical space that are represented by the axioms of Euclidean geometry are better justified than the belief in the Pythagorean character of physical space (represented by the theorem), then the proof (subject only to the assumption that Euclidean geometry is an accurate representation of physical space) can give us more secure knowledge of the Pythagorean character. At the same time it is not clear at all that Pythagoras' theorem is explanatory of this fact regarding physical space.[6] In this way Pythagoras' theorem can make a difference to *how* we know some relevant physical fact, without being *per se* explanatory of any physical fact.

Baker acknowledges that geometrical explanations may not support the second premise of EIA, due to the 'ambiguity between physical and mathematical subject matter of geometry' ([2005], p. 454). It is this (purported) shortcoming that is allegedly overcome by his cicada example that involves abstract number theory instead of geometry. I will next argue that this case is equally undermined by careful attention to the representational role of mathematics as separate from an explanatory role.

[6]The reason (again) is that logical deduction need not track explanation. Euclidean space has many interesting features, and those features are *interdependent* in such a way that a set of features may logically necessitate another feature without explaining it in any standard sense of scientific explanation. It is not at all clear which logical deductions are explanatory and which are not. For example: Pythagoras' theorem follows from Euclid's axioms. The infamous parallel postulate, stating an important feature of Euclidean space, is demonstrably entailed by Pythagoras' theorem. (See, for example, Jordan et al. [1970]) Assume that the Pythagorean character of physical Euclidean space is explained by a proof of Pythagoras' theorem (which necessarily depends on the parallel postulate). Assume that the feature of physical space represented by the parallel postulate is explained by the proof that begins from Pythagoras' theorem (amongst other assumptions). Now, one of the explanans of the Pythagorean character—the parallel postulate—is in turn explained (in part) by Pythagoras' theorem itself! I think the onus is on the advocate of EIA to provide a theory of explanation which makes sense of this kind of explanatory complementarity.

2.2 Cicadas.

Why is the life-cycle period of the North-American cicada exactly 13 or 17 years (depending on sub-species), instead of some other period of 18 years or less?[7] According to Baker ([2005], [2009]), our best scientific explanations of this phenomenon involve indispensable use of number theory and the concept of prime number. Here's Baker's explanatory argument:

(4) Having a life-cycle period which minimizes intersection with other (nearby / lower) periods is evolutionarily advantageous. [biological law]

(5) Prime periods minimize intersection (compared to non-prime periods). [number theoretic theorem]

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(6) Hence organisms with periodic life-cycles are likely to evolve periods that are prime. ['mixed' biological / mathematical law]

(7) Cicadas in ecosystem-type, E, are limited by biological constraints to periods from 14 to 18 years. [ecological constraint]

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(8) Hence cicadas in ecosystem-type, E, are likely to evolve 17-year periods.

What is remarkable here is the alleged explanatory status of *primeness*: the claim that 'it is the link between primeness and minimizing intersection with other

[7] It is important to explicitly specify the *contrast class* in stating the explanandum. Just asking why the life-cycle is 17 years, say, is ambiguous, and receives different answers depending on whether one considers the contrast to be 17 years, or 19 years, for example.

period lengths that does the explanatory work' ([2009], p. 616) In the explanatory argument above this clearly involves premises (5) and (6). Let's replace this injection of mathematics simply by the following:

(5/6)* For periods in the range 14–18 years the intersection minimizing period is 17. [Fact about time]

The conclusion (8) follows immediately, and if Baker's model of explanation captures at all what it is to explain this explanandum, then we have provided an explanation without any number theory in it. The two explanations are not mutually incompatible, but for Baker's argument to go through his mathematics-laden explanation needs to be *more explanatory* than the mathematics-free alternative. I will now argue that this is not the case.

For Baker's explanation to be more explanatory than my alternative, it somehow needs to be a deeper explanation. But to my mind the number theoretic theorem and reference to prime numbers do not further explain the explanatory fact about time expressed by (5/6)*. Rather, the representational capacity of mathematics can be only employed to *justify* our belief in this fact: in as far as mathematics faithfully represents the critical features of time (e.g. its linearity) it can be employed to gain knowledge of facts such as (5/6)*. In this particular case mathematics is not even a privileged method of representation, since we do not need to know any number theory and grasp the concept of prime number to get to know (5/6)*. For example, one could represent periods of time with sticks as follows. Take a bunch of sticks of 14, 15, 16, 17, and 18 centimeters. You'll need less than twenty sticks of each kind. Lay down sticks of each kind one after another to find the least common multiple (LCM) for each pair (viz. the length at which the two lines comprising 14cm and 15cm sticks, say, coincide). You'll soon find out that the least common multiple is almost always clearly longer for the pairs one member of which comprises 17cm sticks.[8] Since linearity of spatial length is taken to faithfully represent the linearity of time, you have discovered (5/6)*. This piece of knowledge is all that is needed to rephrase the explanation by Yoshimura ([1997]). (cf. Baker [2005], p. 454 for a summary of this explanation.)

There is a corresponding fact about time regarding periods in the range 12–14 years: the unique intersection-minimizing period is 13. More generally, a mathematics-free argument with a suitable premise in place of (5/6)* can be given to explain the length of *any particular* cicada period. Furthermore, the respective arguments that explain two different cicada periods will have the same form, and will appeal to same conceptual resources and to same *type* of facts about time, namely, that for some range of periods there is a unique intersection minimizing period. Hence there is a more general (mathematics-free) explanation schema that unifies the particular explanations. What further explanatory worth could mathematics bring into this picture?

Baker thinks that there are two ways in which the explanation involving number theory could go further. Firstly, Baker emphasises the fact that in addition to explaining why the cicada periods are 13 and 17 years some scientists took there to be a *further* explanandum: why are the periods prime?[9] Clearly my mathematics-free alternatives above leave this further explanandum wholly unanswered, as they explain, for example, why the life-cycle of a particular cicada sub-species is 17 years as opposed to 14, 15, 16, or 18 years. Secondly, Baker

[8]The one exception is the following: LCM(14,17)=238, but LCM(15,16)=240.

[9]This is not very explicit in Baker ([2009]), but more so in his ([2005]). Thanks to an anonymous referee for drawing my attention to this.

thinks that the mathematics-hoisted explanation is more explanatory by virtue of being *more general*. He stresses the idea that there is something common to all of these particular explanations—something that can only be captured by injecting mathematics and by appealing the mathematical concept of prime number. That is, from the mathematics-hoisted explanation we allegedly gain *additional* understanding that not only unifies the two cases, but also allows us to project the regularity to possible cases that we haven't yet encountered.

> [T]hese separate [arguments] do not permit any predictions to be made about likely life-cycle durations in other ranges, or for other species. ([2009], p. 617)[10]

I maintain that these (related) reasons are not enough for thinking that prime numbers are doing genuine explanatory work. The problem is that there is again a mathematics-free alternative that unifies the specific explanations of particular period lengths, so the prime-number explanation cannot be viewed as a *deeper* explanation. Again, I admit that the relevant bits of number theory can be knowledge-conferring, but this epistemic virtue can be fully accounted for in terms of mathematics' representational role.

Baker's reasoning runs essentially as follows.[11] We first recognise that there seems to be a unifying fact about the respective life-cycle periods: their lengths (in years) is in both cases prime. This, Baker argues, is best explained by the hypothesis

[10]Baker writes this in reaction to my less measured initial response to him in personal correspondence.

[11]This is extracted from Baker's response to Bangu's criticism which is orthogonal to mine.

> (13) The lengths (in years) of the life-cycles of periodical organisms are likely to be prime [given the number-theoretic explanation that links primeness to minimization of intersection with other period lengths]. ([2009], p. 621)

Furthermore, subject to appropriate ecological constraints the hypothesis (13) also explains the particular life-cycle lengths 13 and and 17. Therefore:

> It is a good explanation because it unifies these two phenomena under a single 'argument pattern', and (relatedly) it can be generalized to other actual or hypothetical cases. For example, it predicts that other organisms with periodical life cycles are also likely to have prime periods. It is therefore better than any historico-ecological explanation that concatenates two separate and independent explanations of the two different period lengths. Hence by inference to the best explanation, we ought to believe in the entities invoked in the number theoretic explanation, which includes abstract mathematical objects such as numbers. ([2009], p. 621)

To my mind this is a rather hopeless attempt to sustain Baker's thesis that 'it is the link between primeness and minimizing intersection with other period lengths that does the explanatory work.' The reason is that we clearly have conceptual resources to frame a mathematics-free explanatory argument pattern that is equally *unifying*, *generalizable*, and *projectible*. To get there, let's modify our earlier explanation of a particular cicada life-cycle to yield the following argument pattern:

(4) Having a life-cycle period which minimizes intersection with other

(nearby / lower) periods is evolutionarily advantageous. [biological law]

(5/6)** There is a unique intersection minimizing period T_x for periods in the range $[T_1, \ldots, T_2]$ years [fact (?) about time]

(7) Cicadas in ecosystem-type, E, are limited by biological constraints to periods from T_1 to T_2 years. [ecological constraint]

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(8) Cicadas in ecosystem-type, E, are likely to evolve T_x-year periods.

This is an argument pattern that unifies the various more specific explanations of this or that cicada life-cycle. It is clearly generalizable to other actual or hypothetical cases. And it can clearly also be used to make predictions. For example, if we discover (by whatever representational means) that 19 years is the unique intersection minimizing period in the range 18–22 years, then we can make the prediction that cicadas would evolve accordingly under the respective ecological constraint.

The premise (5/6)** is either true or false for any given range of periods and some T_x within that range. In order to give a *potential* explanation (along these lines) for why some T_x period has evolved, we must *hypothesise* that (i) there have been appropriate ecological constraints in place, and (ii) (5/6)** really holds with respect to those ecological constraints. In order to argue that our potentially explanation is an *actual* explanation—in order to argue that we *know* why the period is T_x—we need to give evidence for the two hypotheses. The hypothesis (i) is a matter of biology, whilst (ii) states a fact about time. It is at this juncture that mathematics can play a knowledge-conferring role: in as far as mathematics faithfully represents the relevant features of time, we can employ mathematics to demonstrate the truth, or otherwise, of any particular premise of the form (5/6)**.[12]

Finally, I will comment on Baker's appeal to scientific practice. He writes, for example, that

> [Saatsi's view is] in tension with actual scientific practice. Even once biologists had good explanations for the long duration and periodicity of cicada life cycles, they remained puzzled about why these periods have the particular lengths they do. And there is good evidence, based on what they write and say, that this puzzlement only arose because of the fact that both of the known period lengths are prime. ([2009], p. 617)

This is problematic. Although naturalistic philosophy of science should be sensitive to what scientists say about explanation, it is not the case that biologists' pronouncements about the explanatory value of prime numbers needs to be taken at face value. First of all, biologists are not trained to do philosophy, and it is not clear at all what they exactly mean by saying that their explanandum is the fact that cicada periods are prime. Perhaps they use this mathematical language just to express the following explanandum: both cicada life-cycles are intersection-minimising periods. Secondly, even if some biologists did view number theory

[12] I already argued above that mathematics isn't a privileged representational media for any period lengths that are biologically feasible. Using sticks to represent time may be more laborious, of course, but this is besides the point. Also, admittedly there are some facts about time that we can only know via mathematics. For example, consider the fact that in a universe that 'lives' forever there are *infinitely many* unique intersection-minimising periods (each for a particular range). Knowing *this* fact doesn't seem to be of any use when dealing with biological phenomena, but even if it was, the indispensability at issue would only concern the representational use of mathematics.

as part of the explanation, surely it is down to the philosopher of explanation to scrutinise their claims and set them right!

3 Conclusion

We have looked in closer detail at two examples that have been provided in support of EIA's second premise according to which mathematical objects play an indispensable explanatory role in science. We found both of these examples wanting: the indispensable role played by mathematics seems to be purely representational, not explanatory. At the very least an advocate of EIA needs to say significantly more by way of analysing explanation in science to support her claim to the contrary. On a more positive note, our endeavour to separate the representational and the explanatory functions of mathematics is helpful because it indicates how mathematics can play a knowledge-conferring role without being explanatory per se. There are some other alleged examples of explanatory mathematics in the philosophical literature (e.g. Lyon & Colyvan [2008]), and no doubt there are many more to be found in the sciences. Perhaps it will turn out that mathematics does play a genuinely explanatory role in some of these cases, but clearly more needs to be said to substantiate the new wave of the indispensability argument.[13]

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[13] I have severe doubts about the first premise of EIA as well, in as far as the argument is meant to convince the scientific realist that mathematical objects are on a par with unobservables with respect to the realist's commitments. But that's another paper (see Saatsi [2007] for an argument.)

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Department of Philosophy,

University of Leeds

Leeds LS2 9JT, UK

J.T.Saatsi@leeds.ac.uk

References

Baker, A. (2005). Are there genuine mathematical explanations of physical phenomena? *Mind*, 114:223–238.

Baker, A. (2009). Mathematical explanation in science. *British Journal for the Philosophy of Science*, 60:611–633. Forthcoming in BJPS.

Colyvan, M. (2006). Scientific Realism and Mathematical Nominalism: A Marriage Made in Hell. In Cheyne, C. and Worrall, J., editors, *Rationality and Reality: Conversations with Alan Musgrave*, pages 225–37. Springer.

Hales, T. (2001). The Honeycomb Conjecture. *Discrete and Computational Geometry*, 25:1–22.

Jordan, J. Q. and O'Malley, J. M. (1970). An Implication of the Pythagorean Theorem. *Mathematics Magazine*, 43:186–169.

Lyon, A. and Colyvan, M. (2008). The Explanatory Power of Phase Spaces. *Philosophia Mathematica*, 16(2):227–43.

Mancosu, P. (2008). Explanation in Mathematics. *Stanford Encyclopdia of Philosophy*.

Melia, J. (2000). Weaseling Away the Indispensability Argument. *Mind*, 109:455–479.

Melia, J. (2002). Response to Colyvan. *Mind*, 111:75–79.

Nelsen, R. B. (1997). *Proofs without words: exercises in visual thinking*. The Mathematical Association of America.

Saatsi, J. T. (2007). Living in Harmony: Nominalism and the Explanationist Argument for Realism. *International Studies in the Philosophy of Science*, 21:19–33.

Yoshimura, J. (1997). The evolutionary origins of periodical cicadas during ice ages. *American Naturalist*, 149:112–124.