Mathematics and program explanations*

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Lyon [this journal, XX] defends mathematical realism by supporting the notion that mathematics can be (in an appropriate sense) indispensable to some of our best scientific explanations. He argues that some mathematical explanations of empirical facts fit Jackson and Pettit's model of program explanation in a way that brings out the indispensable explanatory role of mathematics. Lyon's attempt is a step in the right direction in terms of its spirit, but not in terms of its letter: it is commendable that Lyon strives to explicitly analyze how mathematics can contribute to some explanations, but unfortunately Jackson and Pettit's model doesn't fit the examples cited. Paying closer attention to the model's details also shows that there's little hope for augmenting it to accommodate any mathematical explanations in science. The reason in brief is that mathematics seems unable to enter into an appropriate 'programming' relationship to lower-level explanatory facts.

The philosophical literature contains a number of interesting, varied examples of scientific explanations that involve mathematics in *some kind of* seemingly essential way. Lyon samples some of these examples by way of responding to what could be called 'the representationalist challenge' to the idea that mathematics *per se* sometimes explains empirical facts.

Representationalist challenge. Mathematics can be indispensable to scientific explaining without being explanatory per se. It can be thus for example by virtue of being indispensable to *expressing* or *representing* non-mathematical facts which themselves do all the explanatory heavy lifting. It can also be indispensable by virtue of playing a confirmatory or knowledge-conferring role vis-a-vis some relevant non-mathematical facts.[1]

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[1] See Melia [2000, 2002] and Saatsi [2011], and for a slightly different take, Daly and Langford [2009].

Lyon responds to this line of thought by arguing that his selection of examples, suitably analyzed, show how mathematics can contribute to explanatory power in a way that goes beyond such broadly representational-cum-epistemic role. Specifically, he argues that mathematics in his examples 'does explanatory work' by 'playing the programming role' in the sense of Jackson and Pettit's model of program explanation. But the fit of the examples to the model is problematic (Objection 1), as is the dialectical use of these examples in responding to the above challenge (Objection 2). Reflecting on the program explanation model in general terms also indicates as questionable the whole idea that the model could be appropriated to mathematical realist ends (Objection 3).

Objection 1. It isn't clear that Lyon's examples fit the program explanation model, *regardless of* what mathematics' contribution is taken to be. Lyon's examples are most naturally construed as explanations of *regularities*, but it is not clear that the model—as presented by Jackson and Pettit—is at all applicable to such explananda that do not concern individual events. And so far as Lyon's examples can be construed as concerning events, it is unclear how exactly the model is meant to capture the examples.

A key feature of the Jackson–Pettit model is the connection it makes between a program explanation and a corresponding causal explanation. A program explanation explains by 'programming for' the availability of a corresponding *process* explanation that refers to some causally efficacious properties. On the basis of this tight connection between a program explanation—referring to a higher-order programming property—and a causal process explanation, Jackson and Pettit construe the higher-order property as causally *relevant* (but not efficacious), and thus also explanatorily relevant.[2] The problem with Lyon's examples is that whilst the essential contrast between a program explanation and a corresponding process explanation is intelligible in the case of *event* explananda, it is much less clear whether the programming idea is at all applicable to *regularities.* Jackson and Pettit's exemplars of explanatory 'programming' always involve an intuitively obvious lower-level cause c of some event explanandum, and an accompanying 'causally relevant' higher-order property C, which is clearly related to c by virtue of the fact that C is 'multiply realizable' in c and (alternative lower-level properties) c', c'', etc. (See Jackson and Pettit [1990].) It is difficult to make

[2] Whether or not program explanations should really be viewed as *causal* explanations is a moot point, but it's worth noting the oddity of applying to blatantly non-causal explanations a model explicitly motivated by the need to make sense of (what at least appear to be) higher-level *causal* explanations.

sense of this relationship between a higher-order programming property and its causal ‘realization basis’ in connection with regularity explananda.

For instance, consider Lyon’s example of the ‘graph theoretic’ explanation of the fact that no one has ever continuously walked over all Königsberg’s seven bridges passing over each bridge only once, and no one ever will.[3] Arguably graph theory is indeed somehow involved in explaining this regularity. (Pincock, 2007) But it is not clear that the Jackson–Pettit model is applicable to this example, since it is difficult to make sense of the notion that there exists any corresponding process explanation of this very explanandum.

We could shift our focus to a more limited explanandum that just summarizes the past failures to cross the bridges in this impossible fashion. There exists a causal history that *amounts* to this explanandum, of course, which perhaps could be construed as an event much extended over time. But it is now unclear exactly how that causal history is meant to provide a causal process explanation of the explanandum, and furthermore one whose availability is in some sense ‘programmed’ by the abstract graph theoretic explanation. In what sense does the complex causal history ‘realize’ the higher-order property that does the explaining in the graph theoretical explanation?

With all Lyon’s examples it is unclear what the lower-level cause of the explanandum exactly is and how it is related to any higher-order property at the level of the ‘program explanation’. More would need to be said about the causally efficacious properties involved to make a convincing case for treating these examples as program explanations at all.

Objection 2. Let’s assume for the sake of the argument that such finessing can be done. There may also be suitable single-event explananda in the vicinity of some of Lyon’s examples, providing more fitting examples.[4] Still, it is not clear why the higher-order property *doing* the programming in these cases should be viewed as a *mathematical* property. That is, it is not clear why mathematics cannot be viewed as playing a broadly representational role vis-à-vis some nominalistically acceptable higher-order property.

For instance, take the lovely example of the mathematical explanation of Plateau’s laws for soap films. Take the mathematical fact that the surface area of such and such geometrical configuration is minimized thus and so. Lyon takes this mathematical fact to play a programming role. The alternative is, of course, to take this bit of mathematics to represent a property of

[3]The explanandum as presented by Lyon is a bit ambiguous between this *universal* regularity and a more restricted ‘regularity’ that just points to the past. It is natural to interpret Lyon as having the universal regularity in mind.

[4]For example, in the soap film example one could take as the explanandum the following event: why did the soap form this particular shape in this instance?

physical space instead, and to take this feature of physical space to (somehow) 'program' for different possible process explanations of this effect.[5]

Lyon briefly considers this worry (§4.2 and 4.3) and responds by stressing 'the role of mathematics in doing the programming'. I will claim below (Objection 3) that the prospects of spelling out this locution are dim. But before we get to that, let's focus on the *dialectical force* of Lyon's response as it stands. The point I wish to make here is that it completely hangs on a metaphysical account of mathematics' programming role, i.e. spelling out how a mathematical property can relate to the lower-level causal properties so as to fulfill the 'programming role' as delineated by the Jackson-Pettit model. Unfortunately no account whatsoever has yet been given of this relation. For this reason Lyon's response, as it stands, begs the question by simply *assuming* that mathematics' role in a program explanation must go beyond representing some higher-order non-mathematical property. Lyon's project is commendable in its spirit: appealing to program explanation is meant to advance the debate in face of the fact that 'everyone agrees that it's not enough that there merely be some mathematics in the explanation.' (Lyon, page XX) But, dialectically, it fails in its letter, for all Lyon has shown is that there can be some mathematics in a program explanation.[6]

Objection 3. Is a suitable metaphysical analysis of mathematics' programming role waiting in the wings? A closer reflection on the nature of the programming relation elicits pessimism about this.

Let's focus on Lyon's key positive claim: 'the mathematics cited [in his examples] is indispensable to the programming of the efficacious properties' (page XX). What does this mean? It seems that it must mean that mathematics is involved as a relatum in the programming relation itself, for the only alternative seems to be that mathematics is merely representing a relevant relatum. The problem now is that it is difficult to conceive of a mathematical property being thus involved in a programming relation.

At the heart of the Jackson-Pettit model is the idea that the instantiation of a programming property *ensures* the instantiation of a relevant causally ef-

[5]Saatsi [2011] discusses in detail how this line of thought can be pressed with respect to the cicada case and the honeycomb case independently of the idea of program explanation.

[6]Here's a different way of making the point. In analyzing what is 'doing the programming' it is not enough to just assess what is indispensable for *providing* a program explanation. 'Programming' in the Jackson-Pettit sense is a relation in the world, whilst 'providing an explanation' is epistemic activity, accomplished by representing the world in certain ways. There can be critical leeway between the two: not all things indispensable for providing an explanation necessarily correspond to things in the world. What's required is a metaphysical analysis of mathematics' programming role.

ficacious property: a programming property is always ‘realized’ by a causally efficacious property. The programming relation, then, is a modal relation between properties. It can take different forms; the only constraint is that the instantiation of the programming property must necessitate—logically, metaphysically, or given the laws of nature—the instantiation of *a* causally efficacious property suitably related to the explanandum at hand. Given this, the crucial question here is: how could a mathematical property ever thus necessitate the instantiation of this or that causally efficacious property somehow ‘realizing’ it? There is no logical connection between a mathematical property and any causally efficacious property, so it would have to be a case of a metaphysical connection, or a law of nature linking the two. What kind of metaphysical connection or law of nature could in this way link a mathematical property to causally efficacious properties? It beats me, and I am not optimistic about the prospects of making sense of a mathematical property entering into a kind of programming relation that respects the essence of the Jackson-Pettit model. For it seems that mathematical properties cannot ensure the instantiation of causally efficacious properties in any realist view of mathematics without some unduly ad hoc metaphysical connection being postulated between the concrete world and mathematical abstracta.

Mathematical properties clearly do not necessitate any causally efficacious properties in a Platonist framework. Mathematics textbooks contain truths about mathematical properties of (say) Platonic surfaces—including those referred to in connection with the explanation of Plateau’s laws—and these properties are instantiated in Plato’s heaven without it being adulterated with any ‘lower-level causal relations’.

Some kind of Aristotelean realism about mathematical properties perhaps allows a little bit more room for linking mathematical and causal properties. Franklin [2009] presents mathematics as a ‘science of quantity and structure’, both construed as universals that can be instantiated in the physical world. Could we not entertain the idea that in this metaphysical framework physical soap films exhibiting Plateau’s laws can be viewed as instantiating the relevant mathematical properties? But, of course, no real soap film actually instantiates the *exact* properties investigated by a mathematical theory of minimal surfaces (e.g. geometric measure theory): what we have are idealized mathematical models of real soap films that ignore forces other than those that keep the film together. More importantly, an explanatorily relevant mathematical property can always be instantiated (in this loose sense) in a physical system without being ‘realized’ in the causally efficacious properties related to Plateau’s laws. (For instance, we might craft a model of soap film from some wire and a sheet of copper, also instantiating the minimal surface area for a given geometric configuration.) Thus, such mathematical property does not *ensure* the instantiation of the relevant causally efficacious properties that ground a lower-level explanation of the shape, and hence it cannot function as a relatum in the programming relation. So, even in this somewhat arcane metaphysical framework it is difficult to make sense of the notion that mathematics in and of itself is involved in programming the efficacious properties.

The only option, it seems, is to say that mathematics is involved in the programming relation, not in and of itself, but as an indispensable part of some kind of physical-cum-mathematical property complex. What is such complex like? How does mathematics get involved in programming via such complex? I have no idea.

In conclusion: some critical metaphysical challenges await anyone wishing to defend mathematical realism along these lines. As indicated above (Objection 2), dialectically speaking the prospects of employing the program explanation model to this end hang by a very slender thread of being able to both spell out and justify an appropriate metaphysics to support Lyon's explanatory intuitions. The alternative is still to view mathematics as playing a broadly representational role in scientific explanations.

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