Mathematics and Explanatory Generality: A Nominalist Analysis

Robert Knowles and Juha Saatsi*

11th January 2018

Abstract

We identify a little-acknowledged but widely shared assumption at the heart of the debate surrounding the enhanced indispensability argument: that explanatoriness is just a matter of faithfully representing explanatory features of reality. We argue that this assumption is mistaken by providing a detailed counterfactual analysis of *explanatory generality*: the explanatory virtue for which mathematics indispensably contributes to in 'distinctly' mathematical explanations. In our analysis mathematics makes such explanations better by rendering non-mathematical explanatory information cognitively easier to grasp. The counterfactual framework of our analysis enjoys independent support in a way that offers a naturalistic, non-question-begging nominalist response to EIA, enabling us yto push past the long-standing impasse that has stifled the EIA debate.

1 Introduction

In this paper we provide a nominalist analysis of the explanatory generality of 'distinctly' mathematical explanations of empirical phenomena. This allows us to push past the clear impasse currently stifling the debate surrounding the 'enhanced' indispensability argument (EIA). It will also fill two significant lacunas in this long-standing debate.

According to EIA (standard) scientific realists should be platonists. This argument is driven by the fact that some of our best explanations of certain empirical facts are 'distinctly' mathematical explanations, which seem to turn on general mathematical facts. (We will recall standard examples of such explanations shortly.) Although alternative, nominalistically kosher explanations can always be

*School of Philosophy, Religion, and History of Science, University of Leeds.

offered, arguably such alternatives are less good qua explanations; hence, a realist who infers to the *best* explanation cannot but accept a realist commitment to whatever the explanatory mathematical facts involve. So goes the most prominent naturalistic argument for mathematical platonism (e.g. Baker 2005; Colyvan 2002, 2013; Lyon 2012).

In response to EIA, nominalists can either deny that mathematical explanations are truly better than nominalistic alternatives, or they can deny that mathematics' indispensable explanatory contribution is ontologically committing. We agree with the near-consensus that the first horn is implausible in the face of the broadly shared judgment that mathematical explanations can be clearly better (qua explanations) than any nominalistically expressible alternative. The second horn has been popular amongst the nominalists, but here the debate has reached an impasse. While the platonists take mathematics' explanatory indispensability as an indication that there exist explanatory mathematical features of reality, nominalists take mathematics to play a merely representational or expressive role for making claims about non-mathematical features of reality that do the 'real' explanatory work. On the one hand, drawing such a distinction between 'really explanatory' and 'merely representational' can only be fairly done on some principled, non-question-begging grounds.[1] On the other hand, the indispensability of mathematics for *providing* an explanation is not enough in and of itself to convince the nominalist that the explanatoriness of this explanation (partly) springs from correctly (re)presenting mathematical features of reality. As many have noted, the debate has here reached a serious impasse (e.g. Baker 2017: 2; Knowles and Liggins 2015: 3403-7). The result is predictable: intuition trading, subtle dialectical manoeuvering, and charges of question-begging.

In this paper, we will break through this impasse in favour of nominalism by addressing two significant lacunas in the debate. One lacuna has to do with the current lack of understanding of how distinctly mathematical explanations are better qua explanations—deeper, more powerful, more explanatory—than nominalistic alternatives. Relatively little has been said about this critical issue apart from the frequent platonist assertions that mathematics is indispensable, in particular, to the *most general* explanations of some empirical phenomena.

Mathematics' connection to explanatory generality has been explicitly drawn by Alan Baker (2017), for example, but the general idea goes back for more than

[1] For example, Saatsi (2016b) draws an interesting distinction between 'thin' and 'thick' explanatory roles, arguing that a nominalist can make sense of mathematics' explanatory indispensability by maintaining that mathematics only plays a 'thin' role of "allowing us to grasp, or (re)present, whatever plays a thick explanatory role" (p. 12) Notwithstanding the conceptual room for Saatsi's distinction, platonists can rightly query why the explanatory superiority of distinctly mathematical explanations is best analysed in these terms.

ten years. For example, Mark Colyvan (2002) defends EIA against Joseph Melia (2000) by noting that mathematics is indispensable for a 'unified approach' to presenting and solving disparate scientific problems, and hence 'genuinely explanatory' (p. 72), since 'unification is linked to explanatory power'. Alan Baker and Colyvan (2011) defend EIA by noting that any nominalized explanation of cicada periods is 'both less general and less robust' (p. 331). Colyvan (2013) defends EIA against Yablo (2013) by arguing that Yablo's approach doesn't do justice to the unifying power of mathematics (p. 1042). Matteo Plebani (2016) objects to Liggins' (2016) response to EIA by arguing that Liggins' approach renders our scientific explanations 'at the wrong level of generality' (p. 553). The notion of 'explanatory generality'—together with cognate notions of 'unification' and 'robustness'—has thus been absolutely central to the platonist reasoning that there exist explanatory mathematical features of reality. The way in which mathematics makes explanations better by making them more general raises a distinct challenge for nominalists: how does mathematics improve explanations, if not by capturing explanatory mathematical features of reality? Call this the **challenge from explanatory generality**.

Making this challenge explicit highlights the two little-noted lacunas we are concerned with: first, nominalists have hitherto not responded to this challenge head-on; second, neither side has provided a detailed, satisfactory account of explanatory generality, and how exactly mathematics contributes to it.[2] We will address both lacunas by analysing explanatory generality in the context of a more general, independently well-motivated *counterfactual framework* for comparing explanatory virtues. The general motivations (independent of the EIA debate) for adopting this framework for analysing distinctly mathematical explanations provide the principled, non-question-begging grounds for our nominalist verdict that drops out of this analysis.

In addition to thus breaking the serious impasse surrounding EIA, our analysis instructively highlights and undermines a common, but seldom recognised assumption made by both sides of the EIA debate regarding explanatoriness in general: that explanatoriness is just a matter of faithfully representing explanatory features of reality. Call this assumption 'explanatoriness through representation' (ETR). Insofar as both sides accept ETR, platonists can challenge the nominalists by reference to mathematical explanations that are judged to be more explanatory than nominalistic alternatives: by ETR, the difference in explanatoriness must be due to the mathematical explanations' representing explanatory features of reality

[2]On the platonist side, Aidan Lyon (2012) has framed the explanatory 'robustness' of mathematical explanations in terms of the 'program explanation' model of Jackson and Pettit (1990). See Saatsi (2012, 2016) for criticism.

that their nominalistic alternatives do not, and nominalists are thereby challenged to say what those features are (if not mathematical features). Our analysis reveals, however, that nominalists need not accept the idea that some of our best, most general explanations make indispensable use of mathematics by virtue of those explanations representing explanatory features of reality that any nominalistic alternatives leave out. Rather, nominalists can maintain that distinctly mathematical explanations are best by virtue of maximising the explanatory information they provide to cognitively limited beings (such as ourselves), which is a function *both* of the amount of such information the explanation provides about relevant worldly features, *and* of the way in which this information is presented. In the explanations we consider, mathematics contributes to explanatoriness via the latter parameter, in a way that is fully compatible with nominalism.

To arrive at this conclusion we will first offer a primer on explanatory generality and distinctly mathematical explanations in §2. We will then provide an independently motivated background theory for analysing explanatory generality in §3. Drawing on this, we will in §4 analyse mathematics' contribution to explanatory generality in distinctively mathematical explanations. In §5, we defend our analysis against three objections, before concluding in §6 that mathematics' explanatory contribution via explanatory generality is not ontologically committing.

2 Explanatory generality and distinctly mathematical explanations

What kinds of explanations does the challenge from explanatory generality involve? There are two desiderata. Firstly, given that this challenge turns on mathematics increasing explanatoriness *in comparison to* nominalistic alternatives, the explanations at stake should allow for a comparison of mathematics-laden and mathematics-free formulations. Secondly, the explanations at stake should make it difficult for the nominalist to identify non-mathematical features of the world as underwriting the increase in explanatoriness (as required by ETR). Both desiderata are met by examples familiar from the EIA literature, such as the number-theoretic explanation of cicada periods, and the graph-theoretic explanation of the non-traversibility of Königsberg's bridges. These explanations can be compared to nominalistic alternatives, yet there is clearly something 'distinctively' mathematical about them (cf. Lange 2013), in a way that suggests they turn on general mathematical facts that transcend physical features.

A simple toy-example illustrates this. A philosopher tries to divide twenty-three distinct ideas evenly among three sections, but keeps failing. Why? Because twenty-three is not divisible by three. This is a 'distinctively' mathematical explanation, since the explanandum is not due to contingent laws of nature (causal or otherwise), but holds with a stronger degree of (mathematical or logical) necessity (Lange 2013).

Such explanations exhibit striking generality, in at least two different senses.[3] First, the explanation clearly has nothing to do with the nature of the things being divided, apart from the stipulation that we are concerned with distinct individuals. Thus, the arithmetical statement at the heart of the explanation equally explains why mother cannot divide twenty-three strawberries evenly among her three children. In this way, the explanation is extremely **topic general**. To achieve this, the explanation must abstract away from what kinds of things are being divided. The arithmetical formulation achieves this admirably.

Secondly, the mathematical explanation is naturally equipped to provide answers to counterfactual questions of the following sort: *what if things were different* so that the philosopher had (say) twenty-one (as opposed to twenty-three) distinct ideas to convey? What if she had fifteen ideas? Three-thousand-and-fifteen? As far as the explanation is concerned, there is nothing special about twenty-three, as opposed to any other positive integer not evenly divisible by three. In this sense, the explanation is extremely **scope general**: it allows us to consider any natural number n as a possible value of the explanans variable quantifying the number of things being divided. The arithmetical formulation readily facilitates this abstraction from what actually happened.

In the context of EIA, these two kinds of generality are distinguished by Baker (2017). However, Baker doesn't offer a detailed analysis, which requires a background theory. We provide a background theory next.

3 Background Theory and Methodology

Mathematics' contribution to explanatory power is best analysed in relation to well-formed ideas about the nature of scientific explanation: figuring out what makes competing explanations *more* or *less* powerful requires a prior understanding of what explanatory power *is*. Accordingly, we will appeal to the counterfactual account of mathematical explanations. This will yield a robust understanding of explanatory generality and allow us to locate mathematics' role as a generator of this explanatory virtue.[4]

[3] Cf. Jansson and Saatsi (forthcoming) and Baker (2017). Here we follow Baker's terminology.

[4] We will not argue here further for the counterfactual account vis-à-vis 'distinctly' mathematical explanations. If one does not like this account, one should provide an alternative analysis of these explanations and mathematics' contribution therein (cf. §6). The secondary message of our paper still stands: explanations' ontological commitments can properly be determined only in the context of a sufficiently well-formed account of explanation.

Before reviewing the counterfactual account, let us pre-empt some worries about our modus operandi. Are we begging the question against the platonist? Are we ruling out by fiat certain kinds of explanatory contributions of mathematics by shoehorning distinctly mathematical explanations into a particular account of explanation?

No. We operate in a naturalistic (Quinean) spirit, appropriate for the dialectic of EIA. By adopting an account of explanation that is well-motivated by recent advances in the philosophy of explanation, we are approaching the debate in the light of all available evidence, including evidence that independently supports this account of explanation. This evidence is not meant to trump opposing considerations regarding the explanatory role of mathematics: we can study mathematics' explanatory contributions with an open mind when considering the evidence adduced in support of EIA. (If the counterfactual account does not capture some convincing example of mathematical explanation, or some clear explanatory virtue, we will need a different account.) Thus, we aim to balance a philosophical account of explanation and its overall support with more specific issues regarding mathematical explanations.

Bringing more evidence to bear on the debate is desirable. A widely acknowledged challenge here is to avoid mere trading of intuitions (as noted, e.g., by Baker 2017: 2). We can achieve this by providing independent evidence for our analysis of crucial notions, such as explanatory generality. Various selling points of the counterfactual account support this, assuming it allows us to make sense of the explanations at stake. We thus profitably connect the ontological issue to broader debates in the philosophy of science by allowing our analysis of mathematics' explanatory contributions to be informed and supported by evidence for the relevant account of scientific explanation.[5]

Moreover, the counterfactual account makes room for the (anti-)nominalism debate. On the one hand, it is a broadly *ontic* account of explanation, so it does not jettison the issue of explanations' realist commitments (cf. Saatsi 2016). Ontic accounts take an explanation's explanatory power to at least partly derive from its latching onto worldly things that bear an objective, explanatorily relevant relation to the explanandum. On the other hand, the counterfactual account is *not* committed ETR, nor the claim that successful explanations wear their commitments on their sleeves (Bokulich 2016, Woodward 2003, Potochnik forthcoming). *Explaining* is a human activity, the goal of which is the provision of explanatory understanding. This introduces epistemic and pragmatic aspects to explanation.

[5] Support for the counterfactual account involves its ability to make sense of various kinds of explanations and of comparative degrees of explanatory power, as we will discuss. It also enjoys naturalistic support from, e.g., cognitive psychology, which we will not discuss here. (See e.g. Buchsbaum et al. 2012.)

Finally, the counterfactual account does not rule out an explanation's having platonistic commitments. It need not render mathematical explanations causal. (This would beg the question against the platonist.) Although initially developed as an account of causal explanation, the counterfactual account has been extended to various non-causal explanations, including those of the distinctly mathematical kind (Reutlinger 2016, forthcoming; Saatsi 2016; Baron et al. 2017; Jansson and Saatsi 2016; French and Saatsi forthcoming). There is also nothing in the counterfactual account that rules out mathematical objects or properties bearing objective, explanatorily relevant relations to physical explananda.

We can now briefly review the counterfactual account. At the heart of the account is the idea that explaining is a matter of providing information of systematic patterns of counterfactual dependence. Explanatory counterfactuals are appropriately *directed* and *change-relating*, capturing objective, mind-independent modal connections that show how the value of the explanandum variable depends on the value of the relevant explanans variable(s). These variables stand for suitably conceptualised and individuated worldly features. Explanatory counterfactuals provide 'what-if-things-had-been-different' information, indicating how the explanandum would have been different, had the explanans been different. Explanation-supporting relations—nomological, causal, or mathematical—between the variables can provide this kind of modal information.

Importantly, an account of explanation built on this idea allows us to capture shared intuitions regarding the comparative virtues of different explanations in terms of the counterfactual information provided by them. If explaining is a matter of providing information that answers *what-if* questions, then it is natural to regard as more powerful those explanations that provide *more* such answers (with respect to a given explanandum). This simple idea has rich and non-trivial consequences regarding the various ways in which explanations can be better or worse. Detailed analyses of explanatory power in this spirit have been provided (see Hitchcock and Woodward 2003, and Ylikoski and Kuorikoski 2010). Here we follow the latter authors, who identify five different aspects of explanatory power: non-sensitivity, precision, factual accuracy, degree of integration, and cognitive salience. The three that will be relevant for our analysis of explanatory generality in §4 are the following.

Non-sensitivity: The range of values that the explanans variables can take without breaking the explanatory relationship. For instance, an explanation of tides in terms of Newton's gravitational law has a considerable degree of non-sensitivity with respect to the specific masses and locations of the sun and the moon: the explanation correctly answers a considerable range of *what-if* questions for non-actual values of these variables. This captures what we called *scope generality* in §1. In the toy-example, this was a matter of the explanation allowing us to consider a range of natural numbers n as possible values of the explanans variable that quantifies the number of things being divided.

Degree of integration: The connectedness of an explanation to other theoretical frameworks. From the counterfactual perspective, such integration is an explanatory virtue when it enlarges the range of *what-if* questions answerable with respect to particular explananda, or makes such questions easier to answer. One way theoretical integration can achieve this is by equipping explainers with new inferential resources (Ylikoski and Kuorikoski 2010). For example, the integration of pressure-wave acoustics to a more general mathematical theory of wave phenomena allowed new *what-if* questions to be asked and answered about sounds waves, increasing explanatory understanding of various sound phenomena (Pierce 1989). In our toy-example, the explanation is integrated into arithmetic, which equips explainers with the inferential resources to easily answer a wide range of *what-if* questions with respect to the particular explananda to which the arithmetic is applied.

Cognitive salience: '[T]he ease with which the reasoning behind the explanation can be followed, how easily the implications of the explanation can be seen and how easy it is to evaluate the scope of the explanation and identify possible defeaters or caveats' (Ylikoski and Kuorikoski 2010: 215). Actual explainers are human beings with limited cognitive capacities, and these limitations partly determine which explanations are more or less explanatory by virtue of differing in their capacity to *enable* explainers (with particular training, background knowledge, etc.) to draw counterfactual inferences for different values of the explanans variables. In our toy example, the arithmetical presentation is highly cognitively salient in this way for anyone equipped with a basic background in arithmetic. A purely logical statement that twenty-three distinct individuals are not divisible into three equinumerous collections would not be cognitively salient in the same way, for example (cf. §4).

4 How Mathematics Contributes to Explanatory Generality

We are now in position to analyse mathematics' contribution to the scope and topic generality of distinctively mathematical explanations. In §4.1, we will argue that mathematics' indispensability in the procurement of *scope* generality is a matter of improving cognitive salience. In §4.2, we will argue that mathematics may not be indispensable at all for achieving *topic* generality; moreover, increasing the topic generality of a particular explanation does not make it more explanatory, unless it thereby increases scope generality. Hence, mathematics' usefulness as an explanatory resource turns out to be matter of improving cognitive salience.

4.1 Scope generality

Consider the arithmetical generality at the heart of the toy-example:

(A) There is no n such that $23/3 = n$.

The numeral '23' represents the number of ideas the philosopher has, and this can be substituted for other numerals representing different 'initial conditions'. The scope generality of the explanation corresponds to its range of applicability with respect to the different numbers of ideas the philosopher might have. Consider an explanation that only captures the dependence of the value of the explanandum variable (successful-division-into-three / unsuccessful-division-into-three) on the value of the explanans variable (number of individuals) for twenty-two to twenty-four individuals. Such an explanation gets right that it is impossible to divide twenty-three things evenly into three, and that the philosopher would succeed if there were twenty-four things to begin with. But these tidbits notwithstanding, the explanation is shallow because it fails to capture the much broader-ranging explanatory regularity at stake. For example, if one philosopher tries to divide twenty-three individuals into three, and another tries to divide three-hundred-and-four, they both fail for the *same fundamental reason*: neither set is evenly divisible by three. We do not have one explanation for one case, and another for the other; we have one explanation, supported by one explanatory regularity, and two different initial conditions.

As discussed in §3, scope generality corresponds to an explanation's *non-sensitivity*. This conception of scope generality naturally rides on the back of the idea—at the heart of the counterfactual account—that explanations work by presenting an explanatory relationship that connects the explanandum to the explanans, showing how the former depends on the latter by virtue of providing true *what-if* information (regarding the state of the explanandum) for at least some non-actual values of the explanans variable. The larger the range of non-actual explanans variable values that are truthfully captured by the explanatory relationship, the less sensitive the application of the explanation is to the actual values of these variables—that is, the more scope-general it is.

With this understanding of scope generality in mind, how does mathematics contribute to it, and in what sense is its contribution explanatorily valuable? (A) allows for a *maximally* scope-general explanation: there is no limit to the number of individuals it can take as the value for the explanans variable. One might think that this in and of itself renders the mathematical explanation most explanatory. We regard this diagnosis as overly simplistic. Rather, we should say that an increase in scope generality is explanatorily valuable only to the extent that it (i) covers situations in which we could reasonably be interested, given the explanandum at stake, and (ii) allows us to grasp the explanatory regularities the explanation tries to capture. In our toy-example, the explanatory generalisation (A) massively overshoots with respect to achieving (i) and (ii). For example, the explanation tells us whether the philosopher would be successful in all situations in which the number of ideas exceeds the number of atoms constituting the earth. These situations are not only bizarre; reflecting on them (in addition to the more reasonable scenarios) provides no further information of explanatory relevance to the explanandum at stake.

If this is right, increasing scope generality can only improve an explanation up to a point, which means *maximising* scope generality is not in and of itself explanatorily valuable. This also means that mathematics is not indispensable for generating a desirable level of scope generality. After all, we can in principle state without mathematics the explanatory dependence between the values of explanandum and explanans variables up to the desired point. Suppose we do this for all possible numbers of ideas up to one exceeding the number of atoms constituting the earth, illustrated by the following:

> **(A′)** If there is exactly one idea, then the philosopher will not be able to divide it evenly among three sections $\textbf{and}_{\mathbf{(2)}}$ if there are exactly two ideas, then the philosopher will not be able to divide them evenly among three sections... $\textbf{and}_{(\mathbf{3} \times \mathbf{10^{50}})}$ if there are exactly 3×10^{50} ideas, then the philosopher will be able to divide them evenly among three sections.[6]

Even though the explanation based on (A′) scopes way beyond any reasonable scenario involving a philosopher writing a paper, and achieves the desired level of scope generality, we agree with the platonists that the explanation turning on (A) is really more explanatory. But we do not think this is due to mathematics further increasing or maximising scope generality.

Rather, the (A)-explanation is better because it achieves the desired level of scope generality *without compromising cognitive salience*. Consider how long and cumbersome the nominalistic explanation would be. Presenting and using such an explanation to answer *what-if* questions concerning particular counterfactual

[6]This can be written, mathematics-free, with first-order logic plus identity.

situations would be unduly difficult. By contrast, (A) makes the same explanatory information transparent, and affords a simple means of answering the desired range of *what-if* questions: just plug in the appropriate numeral and calculate. The explanation turning on (A) also avoids the feeling that, though there is no explanatorily relevant information to be gained by using (A) to answer *what-if* questions concerning numbers of ideas greater than the number of atoms constituting the earth, there is something arbitrary about stopping the explanation there (or anywhere else). But note that none of these virtues that make the (A)-explanation more explanatory are due to (A) containing explanatorily relevant information that cannot be in principle presented without mathematics. Rather, they are due to its presenting the relevant information in a better way. Hence, the indispensable contribution that mathematics makes in relation to scope generality is a matter of improving—indeed maximising—cognitive salience.[7]

(Recall, this is not to deny that the explanation turning on (A) contains more information regarding what would happen if the explanans variable values were different. It clearly does: there is no finite limit to the range of possible explanans variables we can plug into (A). What we deny is that, past a certain, reasonable point, this information is of any explanatory value. In other words, we deny that an increase in scope generality is always, in and of itself, an increase in explanatoriness.)

Let's consider this analysis in relation to a less toy-ish example. North-American periodical cicadas lie dormant for 13 or 17 years (depending on the subspecies). Why the prime numbers? Because an organism which lies dormant for a prime number of years minimises the frequency of overlaps between their emergence and the emergence of nearby periodical predators (assuming these nearby predators have life-cycles of between 2 and 12 years).[8]

In the counterfactual framework, this explanation turns on grasping how the fitness-maximising cicada life-cycles—construed as a variable that can take different values—depends on other biologically relevant variables, such as predator species' life-cycles. Roughly speaking, the explanatory counterfactuals capture the dependence of long-run evolutionary outcomes, regarding specific life-cycle periods, on the existence of predator species with life-cycles of nearby periods, as well as 'ecological constraints' that appropriately limit the range of viable possibilities. These counterfactuals are underwritten by an explanatory generalisation. Math-

[7]From the perspective of the counterfactual account this is unsurprising, and far from being an idiosyncratic feature of mathematical explanations. See, e.g. Ylikoski and Kuorikoski (2010: 214-215), who argue quite generally that an explanation can improve in terms of its cognitive salience without an increase in explanatory information.

[8]This follows Baker (2005); see Wakil and Justus (forthcoming) for worries, which are irrelevant for our argument and analysis.

ematics again comes to play an indispensable role in the formulation of the most powerful explanatory generalisation. To see how, consider the following generalisations, each increasing in scope:

(a) Of time periods 12 to 18 years long, 13- and 17-year periods minimise intersection with all periods shorter than 12 years.

(b) Of time periods 12 to 18 years long, 13- and 17-year periods uniquely minimise intersection with all periods shorter than 12 years, and, of time periods 16 to 22 years long, 17- and 19-year periods uniquely minimise their intersection with all periods shorter than 16 years.

(c) If p is any natural number, then p maximises its least common multiple (LCM) with every $q < p$ iff p is prime.[9]

Explanatory generalisations (a) and (b) are nominalistic, while (c) is not. The actual biological explanation relies on (c). We think (along with the platonists) that the explanation turning on (c) is truly more explanatory than any of those turning on a finite nominalistic generalisation. Why? Note first that mathematics is not necessary for increasing scope generality here: the explanation turning on (b) is more scope-general than the explanation turning on (a), and we can further increase scope generality with explanatory generalisations of the same (nominalistic) kind ad infinitum. Clearly, (c) *maximises* scope generality: there is no limit to the range of possible explanans variables it can take, and so no limit to the range of *what-if* questions it can answer. However, we do not think this is why the explanation turning on (c) is better than any of its nominalistic counterparts with a large enough scope.

To see why, recall that an increase in scope generality is explanatorily valuable only to the extent that it covers situations in which we could reasonably be interested, given the explanandum at stake, and allows us to grasp the explanatory regularities the explanation tries to capture. But we can again increase scope of the explanatory generalisation non-mathematically well beyond its capacity to capture the explanatory regularity at stake, well beyond its capacity to answer *what-if* questions in which biologists could reasonably be interested, and hence well beyond its capacity to contribute information of explanatory relevance. For example, a mathematics-free explanation of cicadas' life-cycles that accommodates every

[9]Prime numbers maximise their LCM with lower integers in the following sense. Given two integers p, q, the highest their LCM can be is pq. The LCM of p and each $x < p$ is px iff p is prime. It is not that a prime number is guaranteed to have a higher LCM with a given lower integer than a nearby non-prime. After all, the LCM of 13 and 5 is 65, while the LCM of 14 and 5 is 70.

duration length up to the age of the universe overshoots in this way. So, the (c)-explanation is not more explanatory because it maximises scope generality.

The (c)-explanation is more explanatory because it achieves the desired level of scope generality *without compromising cognitive salience*. A nominalistic explanation that achieves the desired level of scope generality would be extremely long and unwieldy, so extracting from it answers to *what-if* questions would be difficult, time-consuming, and cognitively opaque to us. In contrast, the number-theoretic rule makes generating answers to *what-if* questions very easy and cognitively transparent. (Laplace's demon might care much less!) It also avoids the feeling that, though there would be no explanatory gain in answering *what-if* questions concerning time periods longer than, say, the age of the universe, there is something arbitrary about stopping the explanation there (or anywhere else). Perhaps there is also something misleading about any such limitation, insofar as it suggests that particular durations of cicada life-cycles are explanatorily relevant, rather than the relationship between the life-cycle durations and the predator life-cycle durations. But none of these benefits are due to (c) presenting explanatory information that cannot be presented nominalistically. They are due to its presenting the relevant information in a better way.

We conclude that mathematics' indispensable contribution to scope generality is a matter of improving cognitive salience. This analysis is not specific to a number-theoretic explanation, and it applies, *mutatis mutandis*, to other well-known examples of distinctively mathematical explanations, such as Euler's graph-theoretic explanation of the non-traversibility of Königsberg's bridges, as we demonstrate in Appendix.

4.2 Topic generality

Recall:

> **(A)** There is no n such that $23/3 = n$.

From (A), it is easy to generate further explanations with radically different subject matters: (A) equally explains why mother couldn't share twenty-three strawberries between her three children, and why we cannot spend all our pocket money (twenty-three pence) on three-pence gobstoppers. Despite the variety of subject matters, when described at the appropriate level of generality, it is clear that these scenarios are structurally similar: they concern attempts to divide twenty-three distinct individuals into three equinumerous collections. It is also clear that the attempts fail for the same reason. To achieve the desired level of generality, we must formulate the explanatory generalisation so that it concerns no physical objects in particular. Arithmetic is good way of achieving this. In the counterfactual framework the explanatory value of this is naturally analysed in terms of *degree of integration*—an explanation's connectedness to a larger theoretical framework—discussed in §3.

Is mathematics indispensable for achieving topic generality? It is not clear that it is. Consider our toy-example again, and (A″), where '*F*' is initially interpreted as 'idea':

> **(A″)** If there is exactly one *F*, then we will not be able to divide it evenly into three collections, $\mathbf{and}_{(2)}$ if there are exactly two *F*s, then we will not be able to divide them evenly into three collections... $\mathbf{and}_{(23)}$ if there are exactly twenty-three *F*s, then we will not be able to divide them evenly into three collections.

We can increase the topic generality of (A″) by increasing the generality of the interpretation we give to '*F*'. For example, we can interpret it as 'idea or strawberry or penny', so that (A″) applies to each of the situations described above. We can even interpret it as 'distinct individual' and thereby formulate a non-mathematical explanation that matches the topic generality of (A). Nevertheless, the explanation turning on (A) is better because it achieves the desired level of topic generality (and scope generality) without compromising cognitive salience (cf. §4.1).

This applies to less toy-ish examples. Consider an example from Baker (2017). Why is it that in fixed-gear bicycles with 14-tooth rear cogs and front cogs with either 47, 48, or 49 teeth, those with 47 in front minimise the wear on their rear tire? The best explanation involves the same mathematics as the cicadas case: of (14, 47), (14, 48), and (14, 49), only the first is a *coprime* pair, and coprime integers maximise their LCM with all lower integers. This means that bikes with 47-tooth front cogs will maximise the number of full pedal-turns required to make the rear cog (and therefore the rear tire) return to its original position, and so are less likely to stop on the same patch of tire.

Since this explanation is structurally similar to the cicada case, we can give a topic general explanation that covers both. Baker (2017) does this by expanding the core of the cicada explanation so as to include (c) below. But mathematics is not really required for this integration, which is achieved by the notion of a *unit cycle*—a domain-neutral means of talking about things exhibiting periodicity, temporal, spatial, or otherwise. So, what is mathematics' contribution? Consider the following three generalisations on which an explanation for cicadas' life-cycle periods can be based, each increasing in topic generality:

(a) Of *temporal* cycles with periods of 12 to 18 years, those with 13- and 17-year periods minimise successive co-occurrences of the same pair of cycle

elements with all cycles with periods shorter than 12 years,
and of temporal cycles with periods of 46 to 49 years, those with 47-year periods minimise successive co-occurrences of the same pair of cycle elements with all cycles with periods shorter than 46 years.

(b) Of *unit cycles* with periods of between 12 and 18 units, those with 13-unit and 17-unit periods minimise successive co-occurrences of the same pair of cycle elements with all unit cycles with periods shorter than 12 units,
and of cycles between 46 to 49 units, those with 47-unit periods uniquely minimise successive co-occurrences of the same pair of cycle elements with all cycles with periods shorter than 46 units.

(c) Any pair of *unit cycles* with periods m and n maximises the gap between successive co-occurrences of the same pair of cycle elements if and only if m and n are coprime.

Since (a) and (b) both contain two corresponding conjuncts, the explanations turning on these explanatory generalisations are equally scope-general. While (a) concerns things that have duration (and exhibit periodicity) in time, (b) concerns linearly additive things with extension of any kind. Hence, (b) can also be used to explain the fact that certain fixed-gear bicycles with 47-tooth front cogs maximise the life of their rear tires, making it considerably more topic-general than (a). Now consider an explanation turning on (c) and assume for the sake of argument that it is better than any explanation built on a finite conjunction like (b). This cannot be because it is more topic-general, since the same level of topic generality is secured by (b). Rather, it is plausible to say that this is because (c)-explanation allows us to achieve the desired level of topic generality, *and scope generality, without compromising cognitive salience*.

This demonstrates that mathematics is not always required to achieve topic generality in distinctively mathematical explanations. However, the demonstration in this case relies on there being linguistic resources to express the generalisation in a way neutral enough to accommodate all the relevant subject matters. We think it is plausible that appropriate linguistic resources are available across the board: the couching of the above generalisations in terms of units (of any magnitude) and distinct individuals (of any kind) points the way. However, we do not have the space to demonstrate that this strategy will generalise. So, for the sake of argument, let us assume that mathematics is sometimes indispensable to expressing an explanation in a maximally topic-general way, and ask what kind of explanatory contribution this kind of theoretical integration makes.

From the counterfactual point of view, topic generality—whether or not achieved with mathematics—only contributes to a *particular* explanation if it enables the answering of more relevant *what-if* questions *concerning that explanation's explanandum*. To see this, note first that there is clearly something odd in thinking that, for example, biologists in possession of the number-theoretic explanation of cicada periods could understand *cicadas* better by reflecting on fixed-gear bicycles. Assuming it is a good one, the biologists' explanation is as powerful as it gets, as far as cicadas are concerned, and no amount of structurally similar applications of number theory enhances it.

Similarly, suppose we have hitherto been unable to explain why mother cannot divide twenty-three strawberries between her three children. By learning the explanatory core of our toy-example, (A), an explanation for why mother fails presents itself, putting us in a position to answer a range of relevant *what-if* questions regarding mother's predicament. We have integrated two disparate structurally similar phenomena to the arithmetical background theory. This is a good thing. But the new *what-if* questions we can now answer (with respect to mother) do not yield any further explanatory understanding of why the philosopher cannot divide twenty-three ideas equally between three sections. From a more global perspective, our stock of answerable *what-if* questions has increased; but this is only because we have formulated a new explanation with its own associated *what-if* questions. In cases such as these, topic generality is clearly very useful, but it does not itself increase the explanatoriness of any of the particular explanations.[10]

Mathematics often contributes to the presentation of a general theory that uniformly treats otherwise disparate subject matters. Integrating different subject matters in this way often allows new *what-if* to be asked and answered, or makes such questions easier to answer, rendering the general theory explanatorily very valuable. To the extent mathematics is indispensable for such integration, mathematics is indispensable to our explanatorily most powerful formal theories and the associated explanatory practices. But, even so, mathematics' usefulness as an explanatory resource is still purely a matter of improving cognitive salience. After all, for any particular explanandum, the range of associated *what-if* questions that a body of theory allows us to answer is a measure of the scope generality of the relevant explanation, and mathematics' involvement in securing a desirable level of scope generality is purely that of preserving cognitive salience (cf. §4.1).

[10]From the perspective of the counterfactual account this is again far from being an idiosyncratic feature of mathematical explanations. See, e.g. Ylikoski and Kuorikoski (2010: pp. 214-5), who offer a general discussion of the virtues of formal unification in science without an increase in explanatory information.

5 Responses to Possible Objections

Cognitive salience is a purely presentational-cum-pragmatic feature of explanations, so it is tempting to conclude immediately that mathematics' contribution to distinctively mathematical explanations does not support platonism. However, we should proceed with caution. As per our methodological preamble (§3), we aim to progress the debate without relying on intuitions, and without begging any questions. In this spirit, we will now consider and rebut three possible objections to our analysis: that we fail to capture a kind of explanatory depth provided by mathematics (§5.1); that our analysis of scope generality is unacceptably interest-relative for the debate at hand (§5.2); and that our analysis of topic generality fails to capture the notion of unification at play in this debate (§5.3).

5.1 Depth over breadth?

One might object that we have unduly focused on explanatory generality in terms of an explanation's scope of application (what we might call *explanatory breadth*), and ignored a no less important form of understanding offered by mathematics: that of picking out a more general feature of reality whose instantiation is responsible for the explanandum (what we might call *explanatory depth*). Take the cicada example. Any nominalistic generalisation stating which durations minimise intersections up to some finite limit presents a series of facts about time, revealing a pattern of dependence between certain time periods and the obtaining of an intersection-minimisation relation between them. It does not explicitly tell us *why* these facts obtain, or why the pattern emerges. One could argue that the number-theoretic generalisation provides this further understanding. Consider the full generalisation in terms of which the cicada explanation is typically expressed:

> **(c′)** If p is any natural number, then p is coprime with every $q < p$ (and hence maximises its LCM with every $q < p$) iff p is prime.

This doesn't just tell us, of each prime number, that it maximises its LCM with each integer lower than it. It arguably tells us why this pattern holds, by showing that each of these particular matters of fact (that p_1 maximises its LCM with each $q < p_1$, that p_2 maximises its LCM with each $q < p_2$, etc.) obtain in virtue of a single, more general fact: that all and only prime numbers are coprime with all lower integers. If this is a genuine explanation, it is a mathematical explanation of a mathematical fact—an *intra*-mathematical explanation; but, it could be argued, it furnishes a deeper understanding of the associated facts about time. Again, we have a pattern of particular matters of fact—that each of a certain set of time periods minimises intersection with each time period shorter than itself—and the idea is that (c′) reveals that each of these particular facts obtains in virtue of a single, more general fact, and thus provides a deeper explanation than any nominalistic alternative. This kind of story can be found in the relevant literature.[11]

It appears that the counterfactual analysis we have presented does not have the resources to capture this kind of depth, and so cannot be relied upon to locate precisely the contribution made by mathematics in achieving it.[12] There are several issues to address here, however. Firstly, there is the issue of whether locating a collection of more specific facts as particular instances of a more general fact really explains them. Secondly, assuming it does, there is the further issue of whether this kind of explanatoriness can legitimately be appealed to in the EIA debate. Finally, assuming that it can be appealed to, familiar questions arise regarding the contribution of mathematics, and, without an independently-motivated framework to guide us, there appears to be no non-question-begging means of answering these questions. We will address each of these in reverse order.

Let us assume for now that the kind of depth outlined above is an explanatory virtue, and it is legitimate to appeal to it in the EIA debate. We need to work out two things: (i) the contribution mathematics makes to the procurement of explanatory depth; and (ii) whether this contribution warrants belief in mathematical objects. Regarding (i), with respect to the cicadas explanation, the contribution made by mathematics here does not appear to be an indispensable one, since the more general fact about time is adequately definable in non-mathematical terms. We can define 'coprime', as it applies to time periods measured in years, as follows: two time periods t_1 and t_2 are *coprime* if, for each shorter time period (except for the shared basic unit), successive concatenations of it will not result in a period congruent to t_1 and a period congruent to t_2 (cf. Rizza 2011). We can then say that a time period t is prime iff t is coprime with all time periods shorter than t. Thus, we can claim that the more particular facts about time hold in virtue of the

[11]Baker (2017: 199) appears to have this in mind when he says that the number-theoretic explanation is *deeper* than mathematics-free alternatives, because it tells us why there are unique intersection-minimising periods within some ranges of periods and not others. Pincock (2015) argues that distinctively mathematical explanations (he calls them *abstract explanations*) explain by identifying physical facts as particular instances of more general, abstract facts. Others highlight cases where the mathematics used to explain what look like very different physical phenomena can be unified under a more general mathematical method, and argue that this suggests there is a deeper, mathematical reason lying behind the relevant physical phenomena (e.g. Colyvan 2002 and Baron, Colyvan, and Ripley 2017). See also §5.3 on unification.

[12]Baron, Colyvan, and Ripley 2017 develop a counterfactual analysis of mathematical explanation with an eye to unifying intra- and extra-mathematical explanations, so their account may offer a means of spelling out this kind of depth counterfactually. Nevertheless, the objections raised in this section suggest that the ability of rival analyses to capture the relevant kind of depth is not in itself a reason to prefer them as analyses of the *scientific explanatory value* of distinctively mathematical explanations. This applies equally to rival counterfactual analyses.

more general fact that all and only prime periods are coprime with all time periods shorter than themselves.

However, if this kind of explanatory depth is widespread in distinctively mathematical explanations, then there is no principled reason to think that the mathematics will always be dispensable. Indeed, that mathematics is indispensable to scientific explanation is a mainstay of the EIA debate. But now familiar moves begin to surface. Nominalists will insist that it is natural to take the more general fact in virtue of which the more specific facts obtain to be physical. This is because it is unclear how particular physical facts might metaphysically depend on how things stand in Plato's heaven (e.g. Saatsi 2012). Moreover, explanatory depth appears to derive from identifying *more specific relations* as instances of *more general relations*, not from identifying more specific relations as instances of *mathematical* relations, so the familiar nominalist gambit is dialectically available: it is mathematical *typology* that is indispensable, rather than mathematical *ontology* (to use Yablo's 2012: 1021 turn of phrase); mathematics serves (indispensably) to characterise *physical* facts at a sufficient level of generality (see also Leng 2013). This story appears to do no damage to the sense that the relevant kind of explanatory depth has been achieved. In response, platonists are likely to see this story as unduly optimistic. Perhaps our pre-scientific philosophical convictions about what can and cannot depend on what do not sit comfortably with scientific practice. But, then, so much the worse for our philosophical convictions.

While we are more inclined to agree with the nominalist here, we recognise that there is very little we could do to move the platonist. Clearly, we are back where we started: an impasse. In line with our running theme, breaking through this impasse will involve providing a detailed analysis of explanatory depth in the context of an independently-motivated framework for comparing explanatory virtues. To our knowledge, no such analysis exists. So, *if* the kind of depth identified here is an explanatory virtue, and *if* it is legitimate to appeal to it in the context of the EIA debate, then an impasse remains. Thankfully, we shall see that there are reasons to doubt both *if*s.

Regarding the legitimacy of appealing to explanatory depth, we think that this is a status that must be earned. As we have stressed, the EIA debate is supposed to have healthy naturalistic credentials. To appeal to the presence of a particular explanatory virtue V in this debate, one had better be able to show that V is operative in science. In other words, one needs to show that, all else equal, scientists are (or should be) more confident about explanations that exhibit more of V, than they are (or should be) about explanations that exhibit less of V. It will not be enough to merely show that scientists find explanations that exhibit V explanatory, and then go on to make the conjecture that V is the reason they do. Baker is guilty of this move, for example:

> I do not know how to demonstrate that the mathematical component is explanatory. On the other hand, I think it is reasonable to place the burden of proof here on the nominalist. The way biologists talk and write about the cicada case suggests that they do take the mathematics to be explanatory, and this provides good grounds, at least *prima facie*, for adopting this same point of view. (Baker 2009: 625; Lyon 2012: 572 is 'inclined to agree'.)

This lip-service to naturalism will not suffice. Granted, it seems right that scientists would consider many explanations involving mathematics more explanatory than their nominalistic counterparts; but that leaves it completely open as to whether that is because they are sensitive to what platonists see as mathematics' contribution to these explanations (cf. Knowles and Liggins 2016: 3404). This general point applies directly to the special case of explanatory depth. Until it can be shown that scientific practice is appropriately sensitive to this feature of distinctively mathematical explanations, appeals to it in support of platonism are not legitimate in the EIA debate.

Further, in light of our own analysis of the scientific value of explanatory breadth, as it manifests in distinctively mathematical explanations (cf. §4), there is reason to doubt that a demonstration of the value of explanatory depth is forthcoming. Our analysis explains why distinctively mathematical explanations are to be preferred over their nominalistic alternatives independently of their exhibiting explanatory depth, and it does so by folding it into a general and independently well-supported theory of scientific explanation. On our story, the place of distinctively mathematical explanations in a broader story about the preferences and standards operative in scientific practice is clear. This analysis sets a high bar. In light of this, the prospects of developing an analysis that gives central importance to explanatory depth, and which rivals our own in its naturalistic credentials, appear dim.

Finally, we turn to the issue of the sense in which explanatory depth is really explanatory. Even if our own analysis of distinctively mathematical explanations captures all that is of *scientific* explanatory value in them, perhaps explanatory depth is genuinely explanatory in some non-scientific, but no less valuable sense. Perhaps it provides us with *metaphysical* understanding. There are a few things to say about this. Firstly, if explanatory depth offers metaphysical understanding, it is still unclear what role mathematics plays its procurement. So, for all the reasons presented above, an impasse looms once again. Secondly, whether one finds explanatory depth metaphysically illuminating will depend on ones background metaphysics. Explanatory depth is supposedly achieved by showing that particular matters of fact are instances of some more general matter of fact. But, someone of a Humean bent, who takes everything to be grounded in local, particular matters of fact, is unlikely to think that the depth appealed to here is explanatory. One could, of course, motivate a metaphysic on which the relevant kind of depth comes out as explanatory. However, this brings us to our final point. Given the naturalistic credentials of the EIA debate, one's reasons for preferring this metaphysic had better be appropriately scientifically informed. More generally, one's reasons for appealing to the provision of metaphysical (as opposed to scientific) understanding in the EIA debate had better be appropriately scientifically informed. In the absence of such scientifically-informed reasons, there is no reason as yet to take an appeal to explanatory depth seriously in the context of the EIA debate.

5.2 Scope generality and interest relativity?

In §3 we argued that the cicada explanation turning on the number-theoretic generalisation offers no explanatory information over and above the information provided by some finite, mathematics-free generalisation, because, given the explanandum at stake, we can only reasonably be interested in considering counterfactual scenarios up to some finite limit. One might object: we are doing ontology here, and it is inappropriate to accept or reject ontological theses on the basis of judgements about what we might (or might not) be reasonably interested in. An explanation can be objectively better than another on a certain metric regardless of whether we should find this difference interesting or not. So, perhaps, the number-theory does yield more explanatory information purely by virtue of its allowing us to answer more *what-if* questions than *any* given mathematics-free alternative.

This objection is dialectically unacceptable. We are supposed to be taking considerations about what is explanatory as a guide to ontology. Though there is evidence that, in many cases, *more* scope-general explanations are to be preferred to less *scope-general* explanations, there is no evidence to suggest that the explanatory value of good formal explanatory frameworks is due to them providing answers to *what-if* questions without bounds. In the absence of such evidence, one cannot dismiss judgements that certain information does not help us understand particular explananda, on the basis of prior judgements about what is or is not ontologically significant. To do so gets the dialectic back to front.

5.3 Topic generality and unification?

In §4, we argued that making an explanation more topic general (without thereby also increasing scope generality or cognitive salience) does not increase its explanatoriness. One might object that we have misunderstood the significance of unification as an explanatory virtue. Granted, rendering an explanation more general (so that its topic-independent 'core' applies elsewhere) doesn't help us to understand the original explanandum better; it does, however, result in a new explanation of a broader class of phenomena that includes both the original and the new explanandum. For example, by formulating the cicada explanation in terms of *unit cycles* and the explanatory-generalisation (UC), arguably Baker (2017) has provided a new explanation of *both* cicada life-cycles and the prevalence of certain fixed-gear bicycle cogs. One can say that the mathematics 'broadens the explanatory landscape rather than improving the original explanation' (Ylikoski and Kuorikoski 2010: 216). We now have a choice as to whether to adopt this more general explanation of this more general phenomenon, or keep the relevant explanations separate. We should, ceteris paribus, prefer a more unified theory of the world; so, we ought to adopt the more topic general explanation.

This line of reasoning is problematic for two reasons. First, it departs from the naturalistic spirit of EIA. There is no evidence that evolutionary biologists would prefer (or have any reason to prefer) an explanation that is general enough to apply to both the cicadas and the fixed-gear bicycles. We suspect that evolutionary biologists would have little interest in the more general explanation, qua evolutionary biologists. If anything, there is reason to think that biologists would prefer a less general explanation that is more grounded in the specific phenomenon they are studying. After all, the more general explanation may fudge important distinctions between the respective domains. In general, making explanations more abstract in order to increase integration between disparate areas of inquiry sometimes results in a loss of important domain-specific detail, resulting in a loss of other explanatory virtues, such as factual accuracy, or cognitive salience (cf. Ylikoski and Kuorikoski 2010: 213-214).

This brings us to the second reason. We suspect that, lurking behind the platonist's argument from unification is an allegiance to a defunct Quinean way of thinking: that we ought to commit to all those things talk of which is indispensable to our simplest, most unified theory of the world. We started by asking whether including mathematics in the cicada explanation makes for a better explanation of cicada life-cycles; now we are asking which explanations ought to comprise our best unified world-view. In other words, the goal posts have moved. The whole motivation for appealing to particular explanations is to avoid resting the case for platonism on the Quinean doctrine of confirmational holism, which is both implausible and highly controversial. Yet, arguing in the above manner clearly relies on this principle. Even if we grant that mathematics is crucial for making our overall worldview as simple and unified as possible, we are only then forced to accept platonism if we also adhere to the view that empirical data confirms all parts of this overarching theory.

6 Conclusion

The current debate on EIA hangs on the question of whether mathematics is *itself* explanatory of empirical phenomena. Platonists have argued for more than a decade that it is by claiming that mathematics makes certain explanations better than nominalistic alternatives by making them *more general*. Nominalists have accepted that mathematics is indispensable for some of our best empirical explanations, but maintained that this is due to the indispensable expressive role played by mathematics. Platonists, on the other hand, maintain that it is difficult to see what (non-mathematical) features of reality could be expressed by the mathematics that is thus indispensable. Both sides in this somewhat deadlocked debate share the critical assumption, ETR, that the source of explanations' explanatory power is purely a matter of its correctly representing explanatory features of reality. Our analysis of mathematics' contribution to the explanatory virtues of mathematical explanations, within the counterfactual framework, shows what's wrong with ETR: explanations can become more explanatory without accurately representing further explanatory features of reality.

According to the counterfactual account, explanatory power is a measure of how much modal explanatory information it provides to an explainer. This is a function both of the amount of such information the explanation provides about the relevant worldly features, *and* of the way in which this information is presented. We take it as obvious that only increases in information about the explanatorily relevant worldly features—objective explanatory dependences—as opposed to changes in how such information is presented, stand a chance of securing further ontological commitments. Our analysis of some of the key exemplars of distinctively mathematical explanation, displaying striking scope and topic generality, shows that mathematics does not yield an increase in this kind of ontologically committing explanatory information. Its role is rather that of improving cognitive salience. From the perspective of the counterfactual account we thus get a clear verdict on the ontological question at stake in EIA: mathematics' indispensable role in empirical explanations provides no reason to believe in mathematical objects.

There is some scope to resist our analysis of mathematics' contribution, e.g. by arguing that there is a further, scientifically kosher dimension of explanatory depth that our analysis doesn't capture (cf. §5.1), or by claiming that mathematics' maximising scope generality can in itself be explanatorily valuable in a way that matters to science (cf. §5.2). Any such claim needs to be accompanied by evidence that avoids the kind of intuition-trading and foot-stomping that we have sought to avoid by working within a well-founded theory of scientific explanation. As far as we can see, the only way to do this would be to motivate an alternative account of mathematics' contribution to distinctly mathematical explanations within an equally well-supported theory of explanation.

Appendix

Here we show how our analysis applies to the much discussed Königsberg explanation.[13] Why is it impossible to make a round-tour of the old Königsberg, crossing each of its seven bridges exactly once? We are looking for explanatory generalisations applicable, e.g., to the bridges at 18th c. Königsberg, connecting four landmasses ('islands'). Let's call a bridge system 'even' *iff* each island has 0, 2, 4, 6, 8, 10, or 12 bridges to/from it. If a bridge system is not even, and has no more than 13 bridges to/from it, call it 'odd'. Now consider the following true explanatory generalisations:

(a) Of all bridge systems connecting exactly four islands, with at most two bridges between any two islands, all and only the even ones are tourable.

(b) Of all bridge systems connecting *up to five* islands, with at most *three* bridges between any two islands, all and only the even ones are tourable.

(c) A bridge system connecting any number of islands is tourable if and only it is *Eulerian*.

Only (a) and (b) above are nominalistically acceptable, but clearly explanatory generalisations of this ilk, expressible in first-order logic, can be extended ad infinitum, by expanding both the number of islands covered and the number of bridges connecting any two islands.[14] In the spirit of §4.1 we maintain that there is no *in-principle* problem with using such generalisation to capture the explanatory dependence between the binary explanandum variable (tourable, non-tourable) and the binary explanans variable (even, odd). For reasons given in §4.1 and §5.2, we maintain that one's explanatory understanding of the tourability of Königsberg's bridges would not get any better by reflecting on possible bridge systems where the number of islands and/or bridges exceeds the number of atoms in Königsberg, for example.

Having said that, clearly (c) underwrites the best explanation. But this is only because (c)-explanation achieves the desired level of scope generality *without compromising cognitive salience*. In this case a nominalistic explanatory generalisation, along the lines of (a) and (b), looks simple on the face of it: of all bridge

[13] Our discussion is adapted from Jansson and Saatsi (forthcoming).

[14] Figuring out that there is a given true generalisation along the lines of (a) and (b) is exponentially (or factorially!) hard work by non-mathematical means. But this is a matter of pragmatics of justification of explanatory generalisations, and does not speak against nominalism (Saatsi 2011).

systems connecting n islands, with at most m bridges between any two islands, all and only the *even* ones are tourable. Defining 'even' by nominalistic means becomes rather long and unwieldy, however. Also, (c) avoids the feeling that there is something arbitrary about stopping the explanation to any particular number of islands and bridges. But none of these benefits are due to (c) presenting explanatory information that cannot be presented nominalistically. They are due to its presenting the relevant information in a better way.

References

Baker, A. (2005). Are there genuine mathematical explanations of physical phenomena. *Mind 114*(454), 223–238.

Baker, A. (2017). Mathematics and Explanatory Generality. *Philosophia Mathematica 25*(2), 194–209.

Baker, A. and M. Colyvan (2011). Indexing and mathematical explanation. *Philosophia Mathematica 19*(3), 323–334.

Baron, S., M. Colyvan, and D. Ripley (2017). How mathematics can make a difference. *Philosophers Imprint 17*, 1–19.

Bokulich, A. (2016). Fiction as a vehicle for truth: Moving beyond the ontic conception. *The Monist 99*(3), 260–279.

Buchsbaum, D., S. Bridgers, D. Skolnick Weisberg, and A. Gopnik (2012). The power of possibility: causal learning, counterfactual reasoning, and pretend play. *Philos Trans R Soc Lond B Biol Sci 367*(1599), 2202–2212.

Colyvan, M. (2002). Mathematics and aesthetic considerations in science. *Mind 111*, 69–74.

Colyvan, M. (2013). Road work ahead: Heavy machinery on the easy road. *Mind 121*(484), 1031–1046.

French, S. and J. Saatsi (Forthcoming). Symmetries and explanatory dependencies in physics. In A. Reutlinger and J. Saatsi (Eds.), *Explanation Beyond Causation: Philosophical Perspectives on Non-Causal Explanations*. Oxford: Oxford University Press.

Hitchcock, C. and J. Woodward (2003). Explanatory generalizations, part II: Plumbing explanatory depth. *Nous 37*, 181–199.

Jackson, F. and P. Pettit (1990) Program Explanation: A General Perspective. *Analysis 50*, 107–17.

Jansson, L. and J. Saatsi (Forthcoming). Explanatory abstractions. *British Journal for the Philosophy of Science*.

Knowles, R. and D. Liggins (2015). Good weasel hunting. *Synthese 192*(10), 3397–3412.

Lange, M. (2013). What makes a scientific explanation distinctively mathematical. *The British Journal for the Philosophy of Science 64*(3), 485–511.

Leng, M. (2013). Taking it Easy: A Response to Colyvan. *Mind 121*(484), 983995.

Liggins, D. (2013). Weaseling and the content of science. *Mind 121*(484), 997–1005.

Liggins, D. (2016). Grounding and the indispensability argument. *Synthese 193*(2), 531–548.

Lyon, A. (2012). Mathematical explanations of empirical facts, and mathematical realism. *Australasian Journal of Philosophy 90*(3), 559–578.

Melia, J. (2000). Weaseling away the indispensability argument. *Mind 109*(435), 455–480.

Pierce, A. (1989). *Acoustics: An Introduction to Its Physical Principles and Applications*. Acoustical Society of America.

Pincock, C. (2015). Abstract explanations in science. *Br J Philos Sci 66*(4), 857–882.

Plebani, M. (2016). Nominalistic content, grounding, and covering generalizations: Reply to grounding and the indispensability argument. *Synthese 193*(2), 549–558.

Potochnik, A. (forthcoming). Eight other questions about explanation. In A. Reutlinger and J. Saatsi (Eds.), *Explanation Beyond Causation: Philosophical Perspectives on Non-Causal Explanations*. Oxford: Oxford University Press.

Reutlinger, A. (2016a). Is there a monist theory of causal and non-causal explanations? *Philosophy of Science*.

Reutlinger, A. (2016b). Is there a monist theory of causal and noncausal explanations? the counterfactual theory of scientific explanation. *Philosophy of Science 83*(5), 733–745.

Reutlinger, A. (Forthcoming). Extending the counterfactual theory of explanation. In A. Reutlinger and J. Saatsi (Eds.), *Explanation Beyond Causation*. Oxford: Oxford University Press.

Rizza, D. (2013). The applicability of mathematics: Beyond mapping accounts. *Philosophy of Science 80*(3), 398–412.

Saatsi, J. (2011). The enhanced indispensability argument: Representational versus explanatory role of mathematics in science. *The British Journal for the Philosophy of Science 62*(1), 143–154.

Saatsi, J. (2012). Mathematics and program explanations. *Australasian Journal of Philosophy 90*(3), 579–584.

Saatsi, J. (2016a). On explanations from geometry of motion. *The British Journal for the Philosophy of Science First online*, doi:10.1093/bjps/axw007.

Saatsi, J. (2016b). On the indispensable explanatory role of mathematics. *Mind First online.*, doi:10.1093/mind/fzv175.

Salmon, W. C. (1984). *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association 1984*, 293–305.

Tallant, J. (2013). Optimus prime: paraphrasing prime number talk. *Synthese 190*(12), 2065–2083.

Wakil, S. and J. Justus (forthcoming). Mathematical explanation and the optimization fallacy. *Philosophy of Science*.

Woodward, J. (2003). 'Experimentation, Causal Inference, and Instrumental Realism'. In H. Radder (ed.), *The Philosophy of Scientific Experimentation*, pp. 87–118. Pittsburgh: University of Pittsburgh Press.

Yablo, S. (2013). Explanation, extrapolation, and existence. *Mind 121*(484), 1007–1029.

Ylikoski, P. and J. Kuorikoski (2010). Dissecting explanatory power. *Philos Stud 148*(2), 201–219.