# Symmetries and explanatory dependencies in physics

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## 1. Introduction

In this essay we will investigate explanations that turn on symmetries in physics. What kinds of explanations can symmetries provide? How do symmetries function as an *explanans*? What philosophical account of explanation can naturally capture commonplace symmetry-based explanations in physics? In the face of the importance and prevalence of such explanations and symmetry-based reasoning in physics, it is striking how little has been written about these issues.[1] It is high time to start examining these hitherto largely ignored questions.

In this paper we will argue that various symmetry explanations can be naturally captured in terms of a *counterfactual-dependence* account in the spirit of Woodward (2003), liberalized from its causal trappings. From the perspective of this account symmetries can function in explanatory arguments by playing a role (roughly) comparable to a contingent initial or boundary condition in causal explanations: a symmetry fact (in conjunction with an appropriate connection between that fact and the explanandum) can contribute to provision of what-if-things-had-been-different information, showing how an explanandum *depends on* the symmetry. That is, symmetries can explain by providing modal information about an explanatory dependence, by showing how the explanandum would have been different, had the facts about the symmetry been different.

Explanatory dependencies of this sort need not be causal. Although the counterfactual-dependence view of explanation is best developed in connection with causal dependence, in recent years this view has been extended to various kinds of non-causal dependencies (e.g. Jansson and Saatsi forthcoming, Reutlinger 2016, Saatsi 2016, Saatsi and Pexton 2013). Our discussion of symmetry explanations is more grist to this mill: many (but not all) symmetry explanations are naturally construed as being non-causal, as we will see. But even if symmetry is not a cause of an explanandum, we may nevertheless be able to regard the explanandum as something that depends in an explanatory way on the symmetry in question. Or so we will argue.

There are alternative accounts of explanation that compete with our counterfactual-dependence perspective, especially in the context of non-causal explanations that are highly abstract or mathematical (Pincock 2007, Pincock 2014, Lange 2013; cf. Jansson and Saatsi forthcoming for discussion). One alternative is to operate in the unificationist tradition of Friedman (1974) and Kitcher (1981, 1989). However, this faces well-known problems, not the least of which concerns the heterogeneity of unificatory practices (see e.g. Redhead 1984). In the case of symmetries in physics in particular, although their unificatory force is

[1] Lange's work on symmetry principles and conservation laws is a notable exception (e.g. Lange 2007, 2012).

obviously connected to their heuristic role (as evidenced through the construction of the so-called Standard Model of particle physics) it is unclear how to cash out the unificatory force beyond that role. Of more current interest is a new approach to non-causal explanations developed by Lange (2007, 2012, 2013), who puts the explanatory weight on the independence of the explanandum from particular laws of nature. Interestingly, Lange has also applied this approach to some central issues concerning symmetry explanations. We will discuss Lange's views in as far as it runs contrary to our counterfactual-dependence account, but we will not attempt a broader assessment of these alternative viewpoints. We shall mainly endeavour to show that a counterfactual-dependence account can naturally deal with various symmetry-based explanations, thereby further supporting the now popular idea that explanations – causal and non-causal alike – provide information about worldly dependence relations that show what is responsible for the explanandum at stake. We will also discuss the extent to which this analysis of symmetry explanations requires us to relinquish the notion that all explanatory dependencies in science are causal. (cf. Skow 2014)

The first order of business is to introduce the key notion, symmetry, and its connection to explanation (§2). The rest of the essay is divided between issues concerning the two basic kinds of symmetries found in science: discrete (§3) and continuous (§4).

## 2. Symmetry and explanation: a toy example

What is *symmetry*, then? In very informal and general terms, the notion of symmetry involves *sameness* (or equivalence) in one respect of *X*, in relation to a *change* (or transformation) in another respect of *X*. What 'sameness in relation to change' exactly consists in is determined by the nature of *X*, the kind of transformation at stake, and in what respect I stays the same in relation to that transformation. Most familiar examples involve geometrical figures, spatial transformations (e.g. rotations), and the sameness of the figure (e.g. with respect to its shape) under those transformations. For instance, an equilateral triangle is thus symmetrical with respect to 120 degree turns. It is also symmetrical in relation to a transformation that reflects or flips the figure with respect to one of the three axes of symmetry.

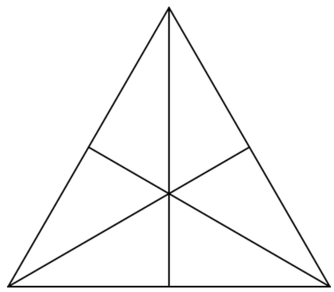

Figure 1: Equilateral triangle with its three axes of symmetry.

More interesting objects of symmetry can involve things like laws of nature (or their mathematical expressions), which can retain their content (or form) under transformations of frames of reference (or coordinate systems). Regardless of the subject matter, symmetry can usually be made precise via the mathematical terms of group theory, where it is naturally defined as *invariance under a specified group of transformations*. The group theoretic framework makes precise the intuitive notion of 'sameness in relation to change' by showing how a symmetry group partitions the object of symmetry into equivalence classes, the elements of which are related to one another by symmetry transformations.[2]

With this notion of symmetry in mind, let's look at a simple toy example of a symmetry, and a related explanation. Consider a balance (a see-saw, say), in a state of equilibrium. Assume the balance remains in the state of equilibrium when particular forces are applied on its two arms. Why does the balance remain in balance? How do we explain this? The standard answer is to appeal to the (bilateral) symmetry of the situation: there is an appropriate equivalence between the forces on the two arms, so that the torque applied from each side to the pivot point is equal – viz. the net torque vanishes. Given this equivalence there are no grounds for the balance to move and hence it remains in equilibrium. Brading and Castellani (2003) call this a 'symmetry argument', and note that the lack of grounds can be understood as an application of the Principle of Sufficient Reason. Our interest lies in, first, the explanatory nature of the argument and secondly, and more importantly, in the role of symmetry as part of the explanans.

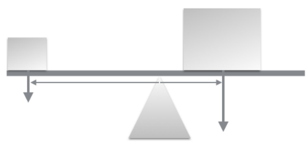

Figure 2. Balance

Let's see how the symmetry argument could be accommodated in the counterfactual-dependence framework, which has at its core the idea that an explanation shows how the explanandum depends on the explanans. Can we find in the case of the balance an explanatory (asymmetric) dependence, associated with counterfactual information that answers what-if-things-had-been-different questions?

The answer is yes: the toy example fits the counterfactual-dependence account of causal explanation. The relevant physics is exceedingly simple, of course. The balance stays in a state of equilibrium if and only if the net torque on the pivot point is zero. This law-like connection between the (non-)equilibrium state of the balance and the forces involved obviously allows us to run the argument in both ways. On the one hand, from vanishing net torque we can deduce the state of equilibrium (assuming the balance was initially at rest). On the other hand, we can also deduce from a state of equilibrium the vanishing net torque. There is no asymmetry inherent in the law we employ in the explanation. (An attempt to

[2] For details, see e.g. Olver (1995).

capture the explanatory symmetry argument in the DN-model thus immediately runs into familiar problems regarding explanatory asymmetry.)

Nevertheless, intuitively there is an obvious explanatory asymmetry to be found: we can change the net torque (by intervening on the forces involved) so as to thereby change the (non-)equilibrium state of the balance, but not the other way around. That is, we cannot change the net torque through somehow acting on the (non-)equilibrium state of the balance, without intervening on the forces involved. That is why is the vanishing net torque is not explained by the equilibrium state of the balance; it is only explained in terms of the forces that 'sum up' to zero. The counterfactual-dependence account of explanation, as developed by Woodward (2003), capitalizes on this explanatory asymmetry. In this case the counterfactual dependence involved has a natural interventionist-causal interpretation, of course. The explanation provides (high-level) information about the causes acting on the balance, and what would happen (vis-à-vis equilibrium) if the forces were different in the relevant ways.

What role does *symmetry* play in the explanation then? Although we are dealing with a causal explanation, there is clearly a sense in which the explanandum depends on a symmetry exhibited by the system. Since *any* non-zero net torque would move the balance to a non-equilibrium state, we can take as the relevant explanans a high-level feature of the system that abstracts away from lower-level information regarding the specific forces applied: all that matters for the explanation is whether or not there is a bilaterally equivalent, *symmetrical* distribution of forces. There is thus a natural sense in which the equilibrium depends on symmetry. Now of course, in this case, there are forces involved, so what we have is a symmetry of such causal factors. Nevertheless, we will next argue that this example of symmetry explanation is not that different from other examples of symmetry based explanations where the existence of such fundamental causal factors is either questionable, at best, or entirely lacking.

## 3. Discrete symmetries

The bilateral symmetry in the toy example above is an example of *discrete* symmetry. These are symmetries represented by groups involving discrete sets of elements (where these elements are typically enumerated by the positive integers). They frequently arise within physics, and include the well-known examples of Permutation Invariance and Charge-Parity-Time symmetry.

Let's begin with Permutation Invariance.[3] To get an idea of what it involves, consider the standard example of two balls distributed over two boxes. Classically, we obtain four possible arrangements, but in quantum mechanics only three arise: both balls in the left hand box (say), both in the right hand box, or one ball in each. The crucial point is that a permutation of balls between the boxes is not counted as giving rise to a new arrangement, and it is upon this exemplification of Permutation Invariance that all of quantum statistics rests. In most textbooks on the subject this is taken to come in just two forms. Bose-Einstein statistics, which – in terms of our simple example – allows for both balls (or particles) to be in the same box (or state), applies to photons, for example. The alternative, Fermi-Dirac statistics, which applies to electrons, for example, prohibits two particles from occupying the

[3] See French and Rickles (2003), and French and Krause (2006), for details.

same state. These two possibilities are encoded in what is generally taken to be a fundamental symmetry of quantum mechanics, captured by the 'Symmetrization Postulate', which says that the relevant wave or state function must be either symmetric – corresponding to Bose-Einstein statistics – or anti-symmetric – generating the Fermi-Dirac form. However, as is well-known, the mathematics of group theory allows for other possibilities, including the statistics of so-called 'paraparticles'.[4] These further possibilities are encoded in a broader principle, known as Permutation Invariance, which, when applied to a particular system, dictates that the relevant Hamiltonian of the system must commute with the group theoretic particle permutation operator (French and Rickles *op. cit.*; French and Krause *op. cit.*).[5] Although parastatistics do not appear in nature (as far as we know)[6] Permutation Invariance is generally regarded as the more fundamental symmetry principle (Greenberg and Messiah 1964).[7]

Now, consider the following as an example of the role of Permutation Invariance in an explanation. Those stars that develop into red giants but have masses less than four times that of the sun (which thus includes the sun itself) will in due course undergo a collapse, until they form a so-called 'white dwarf'. (White dwarves' average diameter is of the order of the Earth's diameter, and they have correspondingly massive density.) The explanation of the collapse has to do with the fact that such stars do not have sufficient energy to initiate the fusion of carbon (their hydrogen having been used up) and thus the balance between the gravitational attraction and the outward thermal pressure is disturbed, in favour of the former. However, there is a further phenomenon that demands explanation: why, at a certain point, does this collapse halt? The answer, given in the physics textbooks, is that this has to do with 'electron degeneracy', understood in this case as the result of the application to stellar statistical physics of Pauli's Exclusion Principle (PEP). The central idea is that according to PEP, no two electrons can be in the same state, and hence as the star contracts, all the lower energy levels come to be filled, so the electrons are forced to occupy higher and higher levels, which creates an 'effective pressure' that eventually balances the gravitational attraction.

The explanation of the halting of the white dwarf collapse thus critically turns on PEP. We regard it as a symmetry-based explanation since PEP, furthermore, drops out of the Symmetrization Postulate, which, we recall, requires the wave functions of all known types of particle to be either symmetric, yielding bosons, or anti-symmetric, corresponding to fermions, which behave according to Fermi-Dirac statistics. It is the latter anti-symmetry that gives rise to PEP. This distinction corresponds to perhaps the most fundamental natural kind

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[4] 'Infinite' statistics are also allowed (Greenberg 1990) and in spaces of less than three dimensions one obtains 'braid' statistics and anyons.

[5] Permutation Invariance thereby divides Hilbert space up into superselection sectors corresponding to the possible types of permutation symmetry associated with the different kinds of particles (bosons, fermions, para-bosons, para-fermions and so on).

[6] Although it was suggested in the mid-1960s that quarks might be paraparticles of a certain statistical type, this was subsequently abandoned in favour of a description in terms of the property that became known as 'colour', leading to the development of quantum chromodynamics (French 1995).

[7] It also grounds the well-known discussions of particle indistinguishability in quantum physics; see French and Krause *op. cit.*.

distinction there is as fermions make up what we might call the 'material' particles, whereas bosons are the 'force carriers'.[8]

It has been suggested that this represents an example of a non-causal explanation of a physical phenomenon, given that Pauli's Principle puts a global constraint on possible states of the system.[9] How should this explanation be understood? A number of philosophers have fretted over this question. Lewis, for example, talks of PEP as representing 'negative information' about causation:

> "A star has been collapsing, but the collapse stops. Why? Because it's gone as far as it can go. Any more collapsed state would violate the Pauli Exclusion Principle. It's not that anything caused it to stop—there was no countervailing pressure, or anything like that. There was nothing to keep it out of a more collapsed state. Rather, there just was no such state for it to get into. The state-space of physical possibilities gave out. … [I]nformation about the causal history of the stopping has been provided, but it was information of an unexpectedly negative sort. It was the information that the stopping had no causes at all, except for all the causes of the collapse which were a precondition of the stopping. Negative information is still information." (Lewis 1986, 222–23)

Attempting to shoehorn this into the causal framework by suggesting that the lack of causal information is still indicative of causal relevance, might strike many as a desperate manoeuvre. Skow (2014), however, has recently argued that it can be brought into the causal framework, insisting, first, that it is not the case that the stopping had no causes at all and second, that there are in fact states for the electrons to 'get into'.

With regard to the first point, Skow notes that many physics textbooks standardly refer to the 'pressure' of a degenerate electron gas in this and other cases. He insists that there is, therefore, a sense in which we can attribute a countervailing pressure to the gravitational attraction, so that the explanation can be regarded as causal. It is important to note, as Skow himself does, that the so-called 'pressure' in this case is very different from that ascribed to a gas, say, since it is not due to any underlying electrostatic force, or indeed any force at all. Indeed, in the years following the establishment of PEP physics struggled to disentangle itself from the understanding of it in terms of 'exclusion forces' and the like (Carson 1996). Thus, one might be inclined to argue that the use of the term 'pressure' here is no more than a façon de parler, or a pedagogic device, and that in terms of our standard conception of pressure as grounded in certain causal features relating to the relevant forces involved (typically electromagnetic), there is simply no such thing as 'degeneracy pressure'.

Skow rejects such a move, insisting that terms in quantum statistical physics, such as 'pressure' and, indeed, 'temperature', have escaped their thermodynamic origins and must be conceived of in more abstract terms than as resulting from the force based interactions of

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[8] But as we also noted, the restriction to only symmetric and anti-symmetric wave-functions is in fact a contingent feature of the world and other symmetry types are theoretically possible, corresponding to para-particle statistics, as permitted by the broader requirement of Permutation Invariance.

[9] Interestingly, physicists never call Pauli's Principle a 'law'. If considered as such, PEP is a *law of co-existence*, as opposed to a *law of succession*. The former restrict positions in the state-space, while the latter restrict trajectories in (through) the state-space. (See van Fraassen 1991, 29.) It is also a *global* constraint that concerns the universe as a whole, not some subsystem of it.

particles or as identical to mean molecular kinetic energy, respectively (*op. cit*. pp. 458-459). Rather, according to Skow these terms should be regarded as dispositional: as the disposition of a system to transfer energy or 'volume', respectively, to another body. Thus, something other than repulsive forces between constituents – such as the consequences of PEP, for example – can contribute to the pressure of a system, rendering the 'degeneracy pressure' explanation causal, after all.[10]

Now we might just pause at this point and wonder whether 'pressure', characterised in such abstract terms, can be understood as appropriately causal. After all, in the case of the white dwarf star, this 'transfer of volume' still does not proceed via any of the known forces and it is unclear how to understand this notion in causal terms. Nevertheless, we shall be charitable and set these issues specific to statistical physics to one side as we believe there are reasons for thinking that non-statistical explanations essentially involving PEP clearly go beyond the causal framework.

Consider, for example, the explanation of chemical bonding. In 1927 Heitler and London explained the bonding in a homonuclear molecule such as $H_2$ by explicitly invoking PEP. It had become evident that the attraction between two hydrogen atoms could not be accounted for in terms of Coulomb forces; the key, as Heitler realised, lay with the so-called exchange integral, previously introduced by Heisenberg, which was something purely quantum mechanical, with no classical analogue (Gavroglu 1995, p. 45). Heitler and London proceeded from the fundamental basis that the electrons were indistinguishable and hence the usual way of labelling them when writing out the relevant wave function had to be re-thought.[11] It then followed that the electronic wave function of the two-atom system had to be written in either symmetric or anti-symmetric form, according to the Symmetrization Postulate. With the electron spins incorporated, PEP dictates that the anti-symmetric form be chosen, with spins anti-parallel. This corresponds to the state of lower energy and attraction

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[10] With regard to Skow's second point, concerning Lewis' claim that the collapse stops because there is no state for the star as a whole to get into, Skow insists that this claim is also false (*op. cit*., pp. 459-460). As he points out, what PEP excludes are states of the star in which more than one electron is in the same quantum state. However, he argues, since there are always infinitely many states available, the electrons never run out of states to get into (because there are always some empty ones available, albeit of high energy), no matter how small the star is. Hence the fact that the star stops collapsing at a certain size has nothing to do with the lack of available states for the electrons to occupy. According to Skow, 'no matter how small the star's radius, the electrons never run out of states because there are infinitely many of them. ' (*op. cit.,* p.460) Thus, the cessation of the star collapse is 'not because a state with a smaller radius is physically impossible, but because the star has reached the radius at which the outward-directed pressure in the star exactly balances the inward-directed gravitational forces. This is a paradigmatically causal explanation.' (*ibid*.) However, we think it is odd to insist that the radius of the star can be disassociated from the availability and occupation of electron states, since it is the latter that determine the former: the higher the energy state, or, putting it somewhat crudely, the further away the energy level is, the bigger the star. Skow is right in that the collapse stops when the star reaches a radius at which the degeneracy 'pressure' balances the gravitational attraction, but given that attraction (i.e. given the mass of the star) PEP ensures that it is impossible for the star to achieve a smaller radius, without a reduction in the number of particles (which is possible through a fusion of protons and electrons into neutrons, via inverse beta-decay). When he insisted that the state-space of possibilities gave out, Lewis was assuming the constraint imposed by the gravitational attraction – under those conditions, and given PEP, for the star to occupy a state corresponding to a smaller radius is a physical impossibility for a star of a given number of fermions.

[11] In effect, the labels have to be permuted and an appropriate wave function then constructed. This permutation of the labels was, at the time, understood as signifying that the particles should not be regarded as individuals, although as it turns out, they can be albeit at a certain (metaphysical) cost; see French and Krause *op. cit*.

is thus understood on the basis of energy minimization. Thus, by deploying the Exclusion Principle chemical valence and saturation could be understood and the ‘problem of chemistry’ solved, or as Heitler put it, ‘Now we can eat chemistry with a spoon!’

This forms the basis of valence bond theory, further developed by Pauling and others, and which is now regarded as complementary to molecular orbital theory. Unlike the former, the latter does not assign electrons to distinct bonds between atoms and approximates their positions via Hartree-Fock or ‘Density Function’ techniques. The former explicitly applies PEP right at the start, to obtain what is known as the Slater determinant, in the case of fermions, where this describes the N-body wave-function of the system, and from which one can then obtain a set of coupled equations for the relevant orbitals. The latter begins with the electron density in 3 spatial coordinates and via functionals of that density reduces the N-body problem of a system with 3N coordinates to one of 3 coordinates only. Again the technique explicitly incorporates the ‘exchange interaction’ due to PEP, and together valence bond theory and molecular orbital theory offer a complementary range of tools and techniques for describing and explaining various aspects of chemical bonding. Despite its name, exchange interaction (also sometimes called exchange force) is best construed as a purely *kinematical* consequence of quantum mechanics, having to do with the possible multi-particle wavefunctions allowed by PEP (or, more generally, Permutation Invariance).

For a specific illustration of the explanatory contribution of this kind of kinematic constraint, consider the solubility of salt. Examining the explanation of solubility brings out its non-causal character. We begin with the formation of an ionic bond between Na+ and Cl-, with the bond-dissociation energy ($E_{diss}$) measuring the strength of a chemical bond the breaking of which is required for substance to dissolve:

$$E_{diss} = E^{+} + E^{-} - \frac{Ke^{2}}{r} + C\frac{e^{-ar}}{r}$$

Here the first term stands for the ionization energy, the second for the electron affinity, the third for the Coulomb attraction, and the fourth describes the energy associated with the so-called ‘Pauli repulsion’, arising from PEP.[12] In this case, perhaps even more clearly than above, the sense of ‘repulsion’ is that of a *façon de parler*. The contribution of this symmetry-based term to the dissociation energy is critical, and it does not have a causal origin unlike the other terms, corresponding to none of the four known forces. Furthermore, there is no equivalent move available here to statistical abstraction, as in the case of quantum statistical ‘degeneracy pressure’.

Before we go on to analyse this explanation, it’s worth noting that examples of PEP-based explanations proliferate: numerous mechanical, electromagnetic and optical properties of solids are explained by invoking PEP, including, indeed, the stability of matter itself.[13] Perhaps in certain scenarios, such as that of the white dwarf collapse, a case can be made that the explanation involved can be accommodated within a broad causal (and, if this is the direction in which one’s metaphysical inclinations run, *dispositionalist*) framework.

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[12] For the Pauli repulsion diagram for salt, see http://hyperphysics.phy-astr.gsu.edu/hbase/molecule/paulirep.html#c1

[13] For a quantum theoretic, PEP-based explanation of stability of matter, see e.g. Dyson and Lenard 1967 and 1968. This was already anticipated by Fowler (1926), who only two years after Pauli’s proposal of his exclusion principle, suggested that PEP explains white dwarves’ stability.

However, in the light of the wide range of explanations of very different kinds of phenomena that turn on PEP (and the Permutation Invariance from which it is derived), we would argue that the recognition of the explanatory role played by this fundamental symmetry motivates a move beyond the causal schema to the framework of counterfactual dependence.

How, then, should we characterise these explanations? Let us begin by recalling that at the heart of the counterfactual-dependence view of explanation is the idea that an explanation proceeds on the back of some form of dependence between that which is described by the explanans and the phenomena captured by the explanandum. Strevens also considers, in this spirit, the example of the halting of white dwarf collapse and the role of PEP within his kairetic approach to explanation:

> 'What relation holds between the law [PEP] and the arrest, then, in virtue of which the one explains the other? Let me give a partial answer: the relation is, like causal influence, some kind of metaphysical dependence relation. I no more have an account of this relation than I have an account of the influence relation, but I suggest that it is the sort of relation that we say "makes things happen".' (Strevens 2008, 178)

Metaphysically one can explicate this dependence in various ways (see French 2014), but what we regard as important with respect to the philosophy of explanation is that it can be cashed out via *counterfactual* dependence and thus can underwrite the appropriate counterfactual reasoning. Explanations, whether causal or non-causal, can be supported by a theory that correctly depicts a space of possible physical states with a sufficiently rich structure, such that it grounds robust reasoning that answers 'what-if- things-had-been-different' questions.[14] Such facts about state-space is precisely what we have in the white dwarf case, as Lewis noted. Similarly, in the explanation of salt's solubility, and in a host of other explanations, PEP imposes a global constraint upon a space of possible physical states, yielding the robust explanatory dependence of the explanandum on the global symmetry. Due to the *global* character of that constraint the relevant counterfactuals are quite different from the interventionist counterfactuals associated with causal explanation. But the spirit of the Woodwardian counterfactual framework still holds.

In the case of PEP, the relevant counterfactuals involving changes in the explanans turn on asking 'what if PEP did not apply?'. Note that what we have here is a 'contra-nomic' counterfactual (lumping laws and symmetries together for these purposes). There are, of course, a number of significant issues associated with how we evaluate such counterfactuals but which we do not have the space to go into here. Instead we shall limit ourselves to explicating it, and answering the question, in the context of our concrete examples.

In the case of the explanation of the solubility of salt, if PEP did not apply, then the crucial ionic bond would not form in the first place and we would not have any salt to begin with! More fundamentally, if PEP did not apply then that would imply that electrons would not be fermions and we would not even have ions of sodium and chlorine because there would not be the constraint that leads to electrons occupying the relevant energy states in the way that underpins ionisation (or, indeed, the formation of atoms!). In the case of the white dwarf, if PEP did not apply – viz. if the particles involved were not fermions – the Symmetrization Postulate dictates that the relevant quantum mechanical wave function must be symmetrised,

[14] See Saatsi (2015) for examples of explanations where the relevant structure of the space of possible states concerns closed loops (holonomies) in state space.

yielding Bose-Einstein statistics. Of course, under that form of statistics the white dwarf collapse would not halt at all; indeed, what we would end up with is a form of 'Bose-Einstein condensate'. For phenomena for which the requirement of symmetric wave functions is appropriate, the symmetrization postulate serves as an explanans for a whole host of different phenomena, from lasers to superconductivity and the 'fountain effect' in liquid helium-4, where very small temperature differences lead to dramatic (and ultimately non-classical) convection effects (see Bueno, French and Ladyman 2002). And we can go further: if we replace the Symmetrization Postulate with the arguably even more fundamental requirement of Permutation Invariance, then, with the possibility of paraparticle statistics, we get a whole host of counterfactuals – indeed an infinite number – rather than just two. Here, quite interesting statistical behaviour emerges if we ask 'what if there were paraparticles of order such-and-such?' for example. Or more generally perhaps, 'what if we have deviations from either Bose-Einstein or Fermi-Dirac statistics?' (see, for example, Greenberg 1992).[15] And we can go further still: as already noted, in spaces of less than three dimensions, one can obtain kinds of particles (or, rather, 'quasi-particles') known as anyons[16], which explain the fractional quantum Hall effect, regarded as representing a new state of matter manifesting so-called 'topological order'.[17]

To sum up, we have argued that in connection with explanations turning on fundamental discrete symmetries such as PEP we can avail ourselves of a counterfactual framework, but drop the requirement of interventions that effectively mark a causal dependence. What distinguishes the kinds of explanations we are concerned with from causal ones is the nature of the explanans. The relevant counterfactuals are theoretically well-formed (in the sense of being grounded in the relevant – mathematically described – physics), and if true they are indicative of dependence relations that hold between various explananda and fundamental symmetries of the world. But these dependence relations are not causal by virtue of involving a *global kinematic constraint* on the available physical states – an explanans for which the notion of intervention seems inapplicable.

We will bring our discussion of discrete symmetries to a close by suggesting that this analysis can also be extended to cases other than Permutation Invariance. One example is the explanation of universality of critical phenomena, which arguably crucially involves a non-causal dependence between specific universality classes, on the one hand, and a discrete symmetry property of the micro-level interactions (the symmetry of the 'order parameter'), on the other. This dependence is brought out by renormalization group analyses of statistical systems (Reutlinger 2016). For another example, consider the so-called CPT Theorem and the explanations that invoke it. The theorem states that all Lorentz-invariant quantum field theories must also be invariant under the combination of charge conjugation (swapping + for – charges and vice versa; i.e. swapping matter for anti-matter), parity reversal (reflection

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[15] So, returning to the example of salt, we might ask, not just 'what if electrons were bosons?', in which case what we call 'matter' would look ad behave very differently indeed (!), but 'what if electrons were paraparticles of some order?'. In that case, not everything would degenerate into a Bose-Einstein condensate and quite interesting statistical behaviour would result. The point is, however, that changing the explanans would yield very different consequences.

[16] As already noted in fn 4, These are described by the 'braid' group which generalizes the permutation group.

[17] Anyons are described as 'quasi-particles' since it remains contested whether they should be regarded as effectively mathematical devices or real; an experiment supposedly demonstrating the latter remains controversial (Camino, Zhou, and Goldman 2005). However, further suggestions have been made involving the experimental manipulation of anyons (see Keilmann et. al. 2011).

through an arbitrary plane or flipping the signs of the relevant spatial coordinates of the system) and time reversal (flipping the temporal coordinate). It has been invoked to prove the Spin-Statistics Theorem, which states that particles that obey Bose-Einstein statistics must have integral spin and those that obey the Fermi-Dirac form must have half-integral spin.[18] Violations of the components of the invariance also feature in scientific and philosophical explanations. For example, violation of CP symmetry has been used to explain the preponderance of matter in the universe, rather than an equal distribution of matter and anti-matter as would be expected. Our hunch is that such explanations also involve assumptions about non-causal counterfactual-dependencies, but we shall not pursue this further here.

## 4. Continuous symmetries

Let's now move on to consider the other significant kind of symmetry found in science, continuous symmetries, and explanations they can support. Continuous symmetries are described by continuous groups of transformations (in particular the Lie groups which cover smooth differentiable manifolds and which underpin Klein's 'Erlangen' programme of systematizing geometry). They are embodied in classical claims regarding the homogeneity and isotropy of space and the uniformity of time, and are accorded fundamental primacy over the relevant laws in the context of Special Relativity, where the Lorentz transformations are effectively promoted to universal, global continuous spacetime symmetries. The extension of such symmetries beyond the space-time context, to the so-called local 'internal' symmetries in the context of fundamental interactions represents one of the major developments in physics of the past hundred years or so, underpinning the so-called Standard Model (see, for example, Martin 2003).

One of the most celebrated explanatory uses of such continuous symmetries appeals to Noether's famous theorem, connecting continuous symmetries to the existence of conserved quantities. The issue of how to interpret that connection has been the subject of some debate. Thus, although many scientists and philosophers regularly speak of conservation laws being explained by symmetries or by Noether's theorem itself, some have challenged this idea. Brown and Holland (2004), for example, point to the two-way nature of Noether's (first) theorem: it not only allows for a derivation of conserved quantities from dynamical symmetries, but equally for the derivation of dynamical symmetries from knowledge of which quantities are conserved:[19]

> "[The] theorem allows us to infer, under ordinary circumstances for global symmetries, the existence of certain conserved charges, or at least a set of continuity equations. The symmetry theorem separately allows us to infer the existence of a dynamical symmetry group. We have now established a *correlation* between certain dynamical symmetries and certain conservation principles. Neither of these two kinds of thing is conceptually more fundamental than, or used to explain the existence of, the other (though as noted earlier if it is easier to establish the variational symmetry group, then a method for calculating conserved charges is provided). After all, the real physics is in the Euler–Lagrange equations of motion for the fields, from which the

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[18] And likewise for parastatistics, since we've mentioned them.

[19] Here we will only focus on Noether's first theorem, which relates conserved quantities to continuous (global) symmetries in Lagrangian dynamics. The second theorem has to do with local symmetries (viz. symmetries that depend on arbitrary functions of space and time; see Brading and Brown 2003).

> existence of dynamical symmetries and conservation principles, if any, jointly spring." (p. 1138)

Lange (2007) concurs that "it is incorrect to appeal to Noether's theorem to secure these explanations", also pressing the point about the theorem's two-way directionality: "The link that Noether's theorem captures between symmetries and conservation laws is (ahem!) symmetric and so cannot account for the direction of explanatory priority." (p. 465) Lange does not conclude that continuous symmetries cannot play an explanatory role, however, as he goes on to provide his own 'meta-laws' account of the modal hierarchy of symmetries and conservation laws with the intention to secure the explanatory priority of symmetries. We will comment on this account in due course, but let's first consider further the two-way directionality of Noether's theorem.

In our view – from the counterfactual-dependence perspective – little hangs on the fact that Noether's theorem represents a *correlation* between symmetries and conserved quantities. After all, most explanations in physics appeal to regularities that can underwrite derivations running in two directions, only one of which may be considered explanatory. (Our toy example in §2 is a case in point, reflecting a point already familiar from explanations of flagpole shadows, pendulum periods, and so on.) What matters, rather, is whether the physics that connects symmetries and conserved quantities can be regarded as uncovering genuine (causal or non-causal) *dependencies* that underwrite explanations in which symmetries function as explanans. If this can be done, then we can regard such dependencies as the source of the explanatory power of continuous symmetries.

This can be done. To show how, we will first recall the relevant theoretical context. (For details, see e.g. Neuenschwander 2011.) Noether's theorem concerns physical systems amenable to a description within Lagrangian dynamics, in which the system can be associated with a Lagrangian: a function of the system's configuration variables and their rate of change. The system's dynamical behaviour over time is such that it minimizes a functional of the Lagrangian over time. For a system in classical mechanics, for instance, this functional is the time integral of the difference between the kinetic and potential energies:

$$J = \int_a^b (K - U)\, dt = \int_a^b L\, dt$$

The requirement that the system's actual dynamics follows a trajectory that minimizes this functional is called Hamilton's principle. The coordinates of this trajectory will satisfy differential equations called Euler-Lagrange equations.

$$\frac{\partial L}{\partial x^\mu} = \frac{d}{dt}\frac{\partial L}{\partial \dot{x}^\mu}$$

In Lagrangian dynamics there are significant connections between symmetries and conserved quantities that flow out directly from the Lagrangian, without at all having to consider Noether's theorem (and the more general connection between conserved quantities and symmetries of the functional). For instance, it is a straightforward corollary of the Euler-

Lagrange equations that canonical momentum $p^\mu$ is constant if and only if $\frac{\partial L}{\partial x^\mu} = 0$.[20] Similarly, it follows directly from the Euler-Lagrange equations that a system's Hamiltonian is constant if and only if the Lagrangian does not explicitly depend on $t$, viz. $\frac{\partial L}{\partial t} = 0$. When the Hamiltonian (formally defined as $H = p_\mu \dot{x}^\mu - L$) can be identified with (the numerical value of) the system's energy, it can thus be seen that energy conservation is connected to symmetry under a time translation.

These elementary connections between continuous symmetries and conserved quantities in the Lagrangian framework can be viewed as special cases of Noether's theorem, which in its full generality need not come into play in deriving the conserved quantities for a given Lagrangian.[21] Based on these connections, mathematical derivations can run in reverse, too, so as to establish symmetries of the Lagrangian from a given set of conserved quantities. Again, these connections in and of themselves say nothing about explanatory priority. In order to get a handle on that we need to consider the modal information provided by the physics. From the perspective of the counterfactual-dependence account, this explanatory priority is underwritten by the fact that in a typical application of these results to a particular system (e.g. the solar system) there is a natural sense in which the conserved quantities *depend on* the features of the system represented by the Lagrangian and its symmetries, but not the other way around. The Lagrangian and its properties reflect the relevant properties of the system being described: kinetic and potential energy functions, and whatever constraints there are to its dynamics. When we consider changes to these features of the system, we consider changing e.g. the spatial distribution of mass or charge, or their quantity. These changes can have an effect on regularities manifested by the system as it evolves over time: different features of the system may become constants of motion, properties whose values are unchanged over time. The point is that there is no way to alter these regularities concerning the system's behavior – these constants of motion – directly as it were, without acting upon the features of the system that determine the system's behavior. And it is the latter that feature in the Lagrangian, the symmetries of which thereby determine the constants of motion in a way that supports explanatory what-if-things-had-been-different counterfactuals.

This asymmetry is best illustrated with a concrete example. For an elementary case, consider a particle moving under a central force. In spherical coordinates $(r, \theta, \varphi)$, the potential energy $U(r)$ of the particle depends only on the radial coordinate $r$, when a spherically symmetric source of e.g. gravitational or electric force field is located at the origin. The kinetic energy function

$$K = \frac{1}{2} m \dot{\boldsymbol{v}}^2 = \frac{1}{2} m \left( \dot{r}^2 + r^2 \dot{\theta}^2 + r \dot{\varphi}^2 \sin^2 \theta \right)$$

feeds into the Lagrangian $L = K - U(r)$. From Euler-Lagrange equations we get as (separate) constants of motion the azimuthal and polar components of the orbital angular momentum: $p_\theta = mr^2 \dot{\theta}$ and $p_\varphi = mr^2 \dot{\varphi} \sin^2 \theta$. This is why the particle's trajectory is constrained to a plane; this regularity about the dynamics depends on the symmetry of the Lagrangian (viz. symmetry of kinetic and potential energy functions).

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[20] Canonical momentum is defined as $p_\mu = \frac{\partial L}{\partial \dot{x}^\mu}$ for each coordinate $x^\mu$ and its coordinate velocity $\dot{x}^\mu$.

[21] Noether's theorem is broader in that it relates conserved quantities to the symmetries of the *functional* (not just the Lagrangian), yielding conserved quantities that are linear combinations of $H$ and $p_\mu$.

Changing the potential energy function, either in its strength (by varying the amount of mass or charge at the center), or in its spatial geometry by breaking the spherical symmetry in favour of some other symmetry, will have effects on the dynamical behaviour of bodies moving under the potential. These effects are reflected also in the regularities of the dynamics captured by the constants of motion. Grasping the connection between these constants of motion and the symmetries of the Lagrangian enables us to answer what-if-things-had-been-different questions such as: What if the source were not spherically symmetrical? What if the source were a spheroid, as opposed to a sphere? What if the spheroid revolved about its minor axis? What if it oscillated in a particular way? From the counterfactual-dependence perspective this kind of modal information is explanatory: it places the explanandum in a pattern of counterfactual dependencies (as Woodward puts it), thus bringing out how the regular aspects of the dynamics captured by the conserved quantities depend on the symmetries.[22]

In this simple example the asymmetry of dependence is amenable to a 'manipulationist' interpretation, given that the notion of intervention is applicable to the relevant features of the central force system that function as the explanans (cf. Woodward 2003). However, it is worth noting that the explanandum is a *regularity*, and it's not clear whether there is a corresponding event explanandum at all. This casts some doubt on whether the explanation in question should really count as causal. (Saatsi and Pexton, 2013) Furthermore, we will now argue that such an interventionist interpretation of symmetry *qua* explanans need not always be available, and even if it is not available, the derivation of conserved quantities from symmetries can nevertheless be explanatory.

In particular, assume that the closed system we are concerned with is the *whole* universe with its dynamical laws, represented via the Lagrangian, exhibiting certain symmetries. We can, again, answer counterfactual questions of the sort '*What if the universe were not symmetrical in this or that way?'*. Answers to such what-if-things-had-been-different questions bring out the way in which particular conservation laws are counterfactually related to the symmetries at stake, even though it is not clear that counterfactuals regarding alternative symmetries can be interpreted in causal terms, with reference to possible manipulations or interventions. The global symmetries of dynamical laws seem intuitively on a par with e.g. the dimensionality of space – a global feature which Woodward once mooted as grounding a non-causal counterfactual-dependence explanation of the stability of planetary orbits (Woodward 2003, §5.9).

One might worry that we do not have a sufficiently solid grasp on the sense of counterfactual 'dependence' between the symmetries of dynamical laws and conservation laws. Why dependence, as opposed to a mere correlation, as Brown and Holland suggest? We think the reason that physicists often give explanatory priority to symmetries over conservation laws has to do with the fact that in analogous applications of Noether's theorem to particular *sub*systems of the universe, such as the central-force system examined above, the explanatory priority is transparent, partly due to the applicability of notions of manipulation and interventions. Explanatory reasoning about the relationship between conserved quantities and

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[22] This is analogous to the connection between a gravitational pendulum's length and its period. For a given pendulum, we can explain a feature of its dynamical behavior over time, namely its period, in terms of its length (and the gravitational potential). But we do not explain the pendulum length in terms of the period, even though the pendulum law allows for its derivation.

symmetries is naturally extended from such subsystems, involving e.g. central or harmonic forces, to symmetries of the laws covering the whole universe. Given the tight connection between conserved quantities and continuous symmetries in the Lagrangian framework – a connection which Noether's theorem captures in highly general terms – we naturally understand and explain conservation laws in terms of symmetries. This provides a non-causal explanation of particular conservation laws, capturing pervasive regularities of dynamical systems.

The explanatory dependence appealed to here need not be a matter of deep metaphysics. Indeed, in as far as our understanding of the counterfactual-dependence analysis of explanation is concerned, this perspective is meant to be compatible with both Humean and non-Humean approaches to the metaphysics of modality and laws. Remember that for the Humean, dynamic and conservation laws alike are just statements of worldly regularities, the special status of which is underwritten by features of the whole global 'mosaic' of particular facts. Understanding the law-like status of those regularities is partly a matter of grasping which features of the mosaic are responsible for that special status. For the regularities involving conserved quantities, the relevant features involve symmetries, statements regarding which would feature as axioms in the relevant formalisation, according to the Best System Analysis of laws. Grasping how those symmetries are responsible for the regularities that conservation laws represent is only a matter of seeing how mosaics with different symmetries would yield different conservation laws. For the Humean there is no deeper metaphysical connection between symmetries and conservation laws: both concern regularities of the mosaic, connected by Noether's theorem. The connection is necessary to a stronger degree of necessity than nomological or causal necessity, and as such comparable to 'distinctly mathematical' explanations. (Lange 2013; Jansson and Saatsi, forthcoming)[23] Admittedly there is much more to be said to elaborate on this sketch, and the nature of conservation laws and symmetries is a largely unexplored area of Humean metaphysics of science.[24]

Alternatively, one can could try to accommodate such symmetries within a dispositionalist approach to modalities and laws. Bird (2007) dismisses symmetries as temporary features of science, to be dropped from our metaphysics as science progresses. And certainly, the prospects for capturing symmetries via the standard stimulus-and-manifestation characterisation of dispositions look dim (see French forthcoming). Nevertheless, one might adapt some of the recently proposed metaphysical devices in this area to articulate an account of how symmetries might be understood as obtaining from a powers based metaphysics (see Vetter 2015). More plausibly, perhaps, if one were to insist on giving modality some metaphysical punch, as it were, one could interpret symmetry principles such as Permutation Invariance as 'encoding', in a sense, the relevant possibilities. By virtue of that, they could then be understood as inherently or, perhaps, primitively, modal. If, further, such principles were taken to be features of the structure of the world, one would reach a position that could be considered a 'third way' between Humean and dispositionalist accounts (French 2014). And of course, on such a view, the role of such principles as the explanans in the kinds of explanations we have considered here would correspond with their ontological priority as such structural features.

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[23] See also Saatsi and Reutlinger (forthcoming) for related point of view on renormalization group explanations.
[24] For a significant exception, see Yudell (2013).

However, our central point about the explanatory character of symmetry explanations is meant to be independent of the metaphysics of modality that underwrites the explanatory counterfactuals that, in turn, answer the relevant what-if-things-had-been-different questions. This is in contrast with Lange (2007), who regards symmetry principles in science as deeper *meta*-laws that constrain the laws there could be: given such meta-laws, the range of possible laws is restricted to those that comply with the symmetry principles in question. Lange motivates this anti-Humean metaphysics of (meta-)laws by drawing on utterances from prominent scientists, such as Feynman:

> "When learning about the laws of physics you find that there are a large number of complicated and detailed laws, laws of gravitation, of electricity and magnetism, nuclear interactions, and so on, but across the variety of these detailed laws there sweep great general principles which all the laws seem to follow. Examples of these are the principles of conservation. All the various physical laws obey the same conservation principles." (Feynman, 1967, pp. 59, 83)

Although we are sympathetic with the naturalistic spirit of Lange's programme, we also see it as potentially question-begging against the competing Humean accounts. From the perspective of a non-governing conception of laws, the idea that laws are *governed* by higher symmetry principles is obviously problematic, to say the least. Furthermore, arguably the Humean has an alternative account to offer, as indicated above. As far as symmetries are employed to explain *particular* law-like regularities, including specific conservation laws, we maintain that this can be captured in the counterfactual-dependence framework.

Admittedly one can ask a deeper question of why various laws are unified in such a way that they are seemingly *governed* by one and the same symmetry principle. (For example, why are Newton's gravitational law and *Coulomb's* law *both* symmetric under arbitrary spatial displacement?) But although answers to this question are probably not amenable to counterfactual-dependence treatment, it seems to us that the question may not have a scientific explanation at all. Lange provides one *metaphysical* answer to it, Humeans offer another, and structural realists yet another. Assessment of the respective vices and virtues of these competing answers is a matter of wholesale comparison of 'metaphysical packages', and must be left for another occasion. Let us just say that appealing to scientists' sense of 'governance' at the level of broad symmetry principles and meta-laws is potentially question begging in the way such appeal has been deemed problematic at the level of laws 'governing' events and regularities (Beebee 2000).

We will not pursue this metaphysical issue further here, but instead comment on Lange's take on Noether's theorem. According to Lange, Noether's theorem is irrelevant for explaining conservation laws. The argument partly turns on the noted symmetry of the theorem, already discussed above, and partly on the fact that "explanations [of conservation laws] were given long before anything resembling Noether's theorem had been even remotely stated" (2007, p. 465) Lange is right to note this, of course, and we also emphasized the fact that in Lagrangian dynamics symmetries can be linked to conserved quantities in straightforward ways that do not demand anything like the full generality of Noether's theorem. Having said this, it seems to us that Noether's theorem is nevertheless explanatorily relevant in the following sense: it functions in a way analogous to an extremely broad-ranging invariant generalization in supporting counterfactual reasoning, by providing a link between symmetries and conservation that enables us to answer what-if-things-had-been-different questions for a maximal range of alternative situations. As such, the explanatory relevance of Noether's theorem is comparable to that of Euler's mathematical proof (regarding the necessary and sufficient conditions for a graph to have a Eulerian circuit) in relation to the impossibility of traversing all Koenigsberg's bridges by crossing each only once. In both cases we could in principle appeal to much more narrow-ranging generalizations connecting the relevant variables, but the respective mathematical theorems have maximal generality. (Cf. Jansson and Saatsi (forthcoming) for related discussion of the Koenigsberg case.)

## 5. Conclusion

We started our discussion of symmetry explanations with an exceedingly simply toy example, a balance remaining in a state of equilibrium, which was explained by a symmetry of the forces involved. The more interesting real-life symmetry explanations discussed thereafter vary in their features, involving: discrete vs. continuous symmetries; local vs. global symmetries; symmetries that are fundamental vs. non-fundamental. Despite this variance, the cases we have discussed are unified in their explanatory character, which, we have argued, is naturally captured in the counterfactual-dependence framework.

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